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# The Fuzzy Approach to Multidimensional Poverty: the Case of Italy in the 90's

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E M B A R G O

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# The Fuzzy Approach to Multidimensional Poverty: the Case of Italy in the 90's

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## 1. Introduction

Most of the methods designed for the analysis of poverty share two main limitations: i) they are unidimensional, i.e. they refer to only one proxy of poverty, namely low income or consumption expenditure; ii) they need to dichotomise the population into the *poor* and the *non-poor* by means of the so called *poverty line*.

Nowadays many authors recognise that poverty is a complex phenomenon that cannot be reduced to the sole monetary dimension. This leads to the need for a multidimensional approach that consists in extending the analysis to a variety of non-monetary indicators of living conditions.

If multidimensional analysis are increasingly feasible as the available information increases, conversely, it was the development of multidimensional approaches that in turn stimulated in many countries the surveying of a variety of aspects of living conditions.

By contrast, however, little attention has been devoted to the second limitation of the traditional approach, i.e. the rigid poor/non-poor dichotomy with the consequence that most of the literature on poverty measurement continues to be based on the use of poverty thresholds.

Yet it is undisputable that so clear cut a division causes a loss of information and removes the nuances that exist between the two extremes of substantial welfare on the one hand and distinct material hardship on the other. In other words, poverty should be considered a *matter of degree* rather than an attribute that is simply present or absent for individuals in the population.

An early attempt to incorporate this concept at the methodological level (and in a multidimensional framework) was made by Cerioli and Zani (1990) who drew inspiration from the theory of Fuzzy Sets initiated by Zadeh (1965). Given a set X of elements $x \in X$, any fuzzy subset A of X will be defined as follows: $A = \{x, \mu_A(x)\}$, where $\mu_A(x)$: $X \rightarrow [0,1]$ is called the *membership function* (*m.f.*) in the fuzzy subset A. The value $\mu_A(x)$ indicates the degree of membership of *x* in *A*. Thus $\mu_A(x) = 0$ means that x does not belong to *A*, whereas $\mu_A(x) = 1$ means that *x* belongs to *A* completely. When on the other hand $0 < \mu_A(x) < 1$, then *x* partially belongs to *A* and its degree of membership of *A* increases in proportion to the proximity of $\mu_A(x)$ to 1.

Cerioli and Zani's original proposal was later developed by Cheli and Lemmi (1995) giving origin to the so called *Totally Fuzzy and Relative* (TFR) approach[1]. Both methods have been applied by a number of authors subsequently, with a preference for the TFR version[2], and in parallel the same TFR method was refined by Cheli (1995) who used it to analyse poverty in fuzzy terms also in the dynamic context represented by two consecutive panel waves.

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[1] Between Cerioli and Zani (1990) and the TFR development, some intermediate contributions are present in the literature (Dagum, Gambassi and Lemmi, 1992; Pannuzi and Quaranta, 1995; Blaszczk-Przybycinska. 1992). Dagum and Costa (2004) have developed an approach similar to TFR leading to the so called Dagum's decomposition (e.g Mussard and Pi-Alperin, 2005).

[2] For instance, Chiappero-Martinetti (2000), Clark and Quizilbash (2003) and Lelli (2001) use the TFR method in order to analyse poverty or well-being according to Sen's capability approach.

From this point on, the methodological implementation of this approach has developed in two directions, with somewhat different emphasis despite their common orientation and framework. The first of these is typified by the contributions of Cheli and Betti (1999) and Betti *et al*. (2004), focusing more on the time dimension, in particular utilising the tool of transition matrices. The second, with the contributions of Betti and Verma (1999, 2002, 2004) and Verma and Betti (2002), has focussed more on capturing the multi-dimensional aspects, developing the concepts of "manifest" and "latent" deprivation to reflect the intersection and union of different dimensions.

In this paper we describe the latest advance of this method (Betti *et al*., 2005) that combines the two developments mentioned above in the form of an *Integrated Fuzzy and Relative* (IFR) approach to the analysis of poverty and social exclusion.

## 2. The Integrated Fuzzy and Relative approach to the analysis of poverty and social exclusion

### 2.1 The conventional income poverty measure ('Head Count Ratio')

Diverse 'conventional' measures of monetary poverty and inequality are well-known and are not discussed here. In this paper we will focus on only the most commonly used indicator, namely the proportion of a population classified as 'poor' in purely relative terms on the following lines. To dichotomise the population into the "poor" and the "non-poor" groups, each person i is assigned the equivalised income yi of the person's household.[3] Persons with equivalised income below a certain threshold or poverty line (such as 60% of the median equivalised income) are considered to be poor (assigned a poverty index, say, $H_i$=1), and the

[3] Equivalised income is defined as the net disposable total household income discounted to take into account variations in household size and composition. For numerical applications in this paper, we have used the 'modified-OECD' scale.

others as non-poor (assigned a poverty index $H_i$=0). The conventional income poverty rate (the Head Count Ratio, H) is the estimated population average of this poverty index, appropriately weighted by sample weights (wi).

**2.2 The propensity to income poverty ('Fuzzy Monetary')**

In what has been called the "Totally Fuzzy and Relative" approach, Cheli and Lemmi (1995) define the m.f. as 1- F($y_i$), where F( ) is the distribution function of income. According to this definition the degree of income poverty of any individual is equal to the proportion of people who are better off than she is and it turns out to be 1 for the poorest and 0 for the richest person in the population.

Since the mean of the m.f. so defined is always 0.5, by definition, which makes it difficult to compare the results of the fuzzy analysis with the conventional poverty rates, Cheli (1995) takes the same m.f. raised to some power $\alpha \geq 1$:

$$\mu_i = (1 - F_i)^{\alpha} = \left( \frac{\sum_{j=i+1}^{n} w_j}{\sum_{i=1}^{n} w_i} \right)^{\alpha} \tag{1}$$

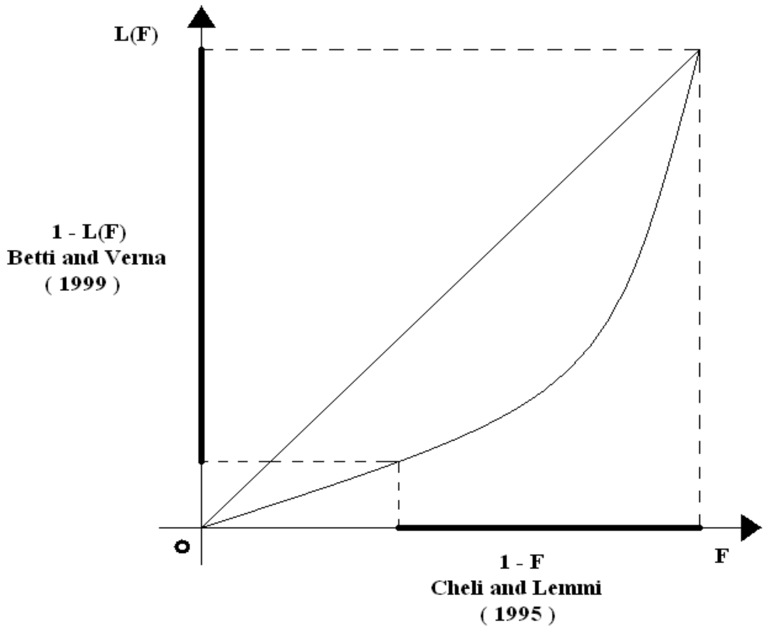
Increasing the value of this exponent implies giving more weight to the poorer end of the income distribution: empirically, large values of the m.f. would then be concentrated at that end. Beyond that, the choice of the value of $\alpha$ is essentially arbitrary, or at best based on some external consideration: this is unavoidable since any method for the quantification of the extent of poverty is inevitably based on the arbitrary choice of some parameter (Hagenaars, 1986). Cheli and Betti (1999) and Betti and Verma (1999) have chosen the parameter $\alpha$ so that the mean of the m.f. is equal to head count ratio H computed for the official poverty line. In this way we avoid an explicit choice of $\alpha$, by adapting to the political choice which is implicit in the poverty line. Moreover, this is to facilitate comparison between the conventional and fuzzy measures.

Betti and Verma (1999) have used a somewhat refined version of the above formulation (1) in the following form in order to define the Fuzzy Monetary indicator (FM):

$$\mu_i = FM_i = \left(1 - L_{(M),i}\right)^{\alpha} = \left( \frac{\sum_{j=i+1}^{n} w_j y_j}{\sum_{i=1}^{n} w_i y_i} \right)^{\alpha}; \qquad (2)$$

where $y_j$ is the equivalised income of individual of rank j in the ascending income distribution and $L_{(M),i}$ represents the value of the Lorenz curve of income for individual i. In other terms, $(1-L_{(M),i})$ represents the *share* of the total equivalised income received by all individuals less poor than the person concerned. It varies from 1 for the poorest, to 0 for the richest individual. $(1–L_{(M),i})$ can be expected to be a more sensitive indicator of the actual disparities in income, compared to the normalised distribution function $(1-F_i)$ which is simply the *proportion* of individuals less poor than the person concerned.

**Figure 1. Membership functions used by Cheli and Lemmi (1995), and Betti and Verma (1999)**



This is illustrated in Fig.1 where the two m.f. specifications are compared (for $\alpha$=1) by means of the Lorenz diagram. It may be noted that while the mean of (1-$F_i$) values for $\alpha$=1 is ½ by definition, the mean of (1-$L_{(M),i}$) values equals (1+G)/2, where G is the Gini coefficient of the distribution.

In a recent contribution (Betti *et al.*, 2005), we have proposed a new poverty measure that combines the two described above and represents a generalisation of both Specifically, the measure is defined as:

$$\mu_i = FM_i = (1-F)^{\alpha-1}.[1-L(F)] = \left(\frac{\sum_\gamma w_\gamma \mid y_\gamma > y_i}{\sum_\gamma w_\gamma \mid y_\gamma > y_1}\right)^{\alpha-1} \cdot \left(\frac{\sum_\gamma w_\gamma y_\gamma \mid y_\gamma > y_i}{\sum_\gamma w_\gamma y_\gamma \mid y_\gamma > y_1}\right), \quad (3)$$

where, again, parameter $\alpha$ is chosen so that the mean of the m.f. is equal to the official head count ratio H:

$$E(FM) = \frac{\alpha + G_\alpha}{\alpha.(\alpha+1)} = H \quad (4)$$

As we see from the previous expression, the mean of the Fuzzy Monetary measure is expressible in terms of the generalised Gini measures $G_\alpha$, which corresponds to the standard Gini coefficient for $\alpha$=1 and is defined (in the continuous case) as:

$$G_\alpha = \alpha(\alpha+1)\int_0^1 \left[(1-F)^{(\alpha-1)}(F-L(F))\right]dF \; . \quad (5)$$

This measure weights the distance $[F-L(F)]$ between the line of perfect equality and the Lorenz curve by a function of the individual's position in the income distribution, giving more weight to its poorer end.

### 2.3 Constructing indicators of non-monetary deprivation

In addition to the level of monetary income, the standard of living of households and persons can be described by a host of indicators, such as housing conditions, possession of durable goods, the general financial situation, perception of hardship, expectations, norms and values. Quantification and putting together of a large set of non-monetary indicators of living conditions involves a number of steps, models and assumptions.

Firstly, from the large set which may be available, a selection has to be made of indicators which are substantively meaningful and useful.

Secondly, it is useful to identify the underlying dimensions and to group the indicators accordingly. Taking into account the manner in which different indicators cluster together (possibly differently in different national situations) adds to the richness of the analysis; ignoring such dimensionality can result in misleading conclusions. Putting together categorical indicators of deprivation for individual items to construct composite indices requires decisions about assigning numerical values to the ordered categories and the weighting and scaling of the measures.

Individual items indicating non-monetary deprivation often take the form of simple 'yes/no' dichotomies (such as the presence or absence of enforced lack of certain goods or facilities). However, some items may involve more than two ordered categories, reflecting different degrees of deprivation. Consider the general case of c=1 to C ordered categories of some deprivation indicator, with c=1 representing the most deprived and c=C the least deprived situation. Let ci be the category to which individual i belongs. Cerioli and Zani (1990), assuming that the rank of the categories represents an equally-spaced metric variable, assigned to the individual a deprivation score as: $d_i = (C - c_i)/(C-1)$, $1 \le c_i \le C$. (6)

Cheli and Lemmi (1995) proposed an improvement by replacing the simple ranking of the categories with their distribution function in the population: $d_i = \{1 - F(c_i)\}/\{1 - F(1)\}$. (7)

Note that the above two formulations for $d_i$ are identical in by far the most common case – that of a dichotomous indicator (C=2), giving a dichotomous m.f. $d_i$= 1 (deprived) or $d_i$= 0 (non-deprived).

The procedure for aggregating over a group of item is also the same for the two formulations: a weighted sum is taken over items (k): $\mu_i = \Sigma w_k d_{k,i} / \Sigma w_k$, where the $w_k$ are item-specific weights, taken as $w_k = \ln(1/\bar{d}_k)$. For dichotomous indicators, $\bar{d}_k$, the mean of individual values ($d_i$) for item k, simply equals the proportion of deprived of that item. The

IFR approach that we developed uses the above framework with some important refinements proposed by Betti and Verma (1999, 2002, 2004) that consist in the construction of non-monetary indicators in exactly the same way as the income indicator described in Section 2.2.

(1) We begin by selecting the items to be included in the index or indices of deprivation on substantive grounds, and grouping the items into 'dimensions'. Deprivation scores ($d_{k,i}$) are assigned to ordinal categories of each item as in (6).

(2) The weights to be given to items are determined within each dimension (group of items) separately as described below in Section 3.3. With these weights, a deprivation score is determined for each dimension ($\delta$: 1,..., $\Delta$): $S_{\delta,i} = \Sigma_{k\in\delta} w_k \left(1 - d_{k,i}\right) / \Sigma_{k\in\delta} w_k$ , and also for the overall situation covering all the indicators: $S_i = \Sigma_k w_k \left(1 - d_{k,i}\right) / \Sigma_k w_k$ .

Note that S is a 'positive' score indicating lack of deprivation; thus it is akin to income in Section 2.2.

(3) As in the Fuzzy Monetary approach, we may consider three alternative definitions for the individual's degree of non-monetary deprivation $FS_i$. All of them are consistent with a relative concept of deprivation and are analogous to the three shapes specified for monetary deprivation.

Restricting our attention to the set of all indicators, for any individual j, $FS_i$ can be defined as:

i) The proportion of individuals who are less deprived than i: $\mu_i = FS_i = \left(1 - F_{(S),i}\right)^{\alpha}$, where $F_{(S),i}$ represents the distribution function of S evaluated for individual i.

ii) The share of the total non-deprivation S assigned to all individuals less deprived than i: $\mu_i = FS_i = \left(1 - L_{(S),i}\right)^{\alpha}$, where $L_{(s),i}$ represents the value of the Lorenz curve of S for individual i calculated according to the form below[4] :

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[4] This forms has been chosen so as to take into account tied rankings, which are much more frequent for items with a few categories, compared to the case of continuous variables like income. This formulation differs from (8) above in that it can be expected to be somewhat more sensitive to

$$1 - L_{(s),i} = \frac{\sum_{\gamma} w_{\gamma} S_{\gamma} \mid S_{\gamma} > S_i}{\sum_{\gamma} w_{\gamma} S_{\gamma} \mid S_{\gamma} > S_1}$$

As the population size tends to infinity, both i) and ii) vary from 0 for the least deprived individual to 1 for the most deprived one, and both of them can be raised to an exponent α >1 that represents the weight of the poorer people compared to the less poor ones.

iii) A combination of the previous two forms similar to formula (5):

$$\mu_i = FS_i = \left[1 - F_{(S),i}\right]^{\alpha-1}\left[1 - L_{(S),i}\right]; \ \ \alpha \geq 1. \qquad (8)$$

We are quite confident that in practice adopting any one of the three specifications above would lead to very similar conclusions. Nevertheless, according to the same theoretical considerations made in the case of income, we prefer to opt for specification iii).

## 2.4 Weights for the aggregation over items

The weights $w_k$ given to the items in the construction of the S scores are specified according to two principles:

a) any weight should increase with the diffusion of the corresponding item, so as to give more importance to the items that are more representative of the lifestyle prevailing in society.

the weighting system should avoid redundancy, that is limiting the influence of those indicators that are highly correlated. In particular the weight of any item is defined as[7]: $w_k = w_k^a * w_k^b$ (9)

where $w_k^a$ only depends on the distribution of item k, whereas $w_k^b$ depends on the correlation between k and other items.

In particular, $w_k^a$ is determined by the variable's power to differentiate among individuals in the population, that is, by its dispersion. In practice

the actual levels of deprivation than the normalised distribution function ($1\text{-}F_j$), the latter being only the *number* of individuals less deprived than the person concerned.

[5] Or the mean value of deprivation if item k is not dichotomous

[6] Such specification was used among the others by Betti *et al.* (2002).

[7] Betti and Verma (1999).

we take this as proportional to the coefficient of variation of deprivation score $d_{k,i}$ for the variable concerned: $w_k^a \propto cv_k$ (10)

This means that the weight varies inversely to the square-root of the proportion. Thus deprivations which affect only a small proportion of the population, and hence are likely to be considered more critical, get larger weights; while those affecting large proportions, hence likely to be regarded as less critical, get smaller weights. Note, however, that the contribution of the deprived individuals to the average values of deprivation in the population resulting from the item concerned turns out to be directly proportional to the square-root of d. In other words, deprivations affecting a smaller proportion of the population are treated as more intense at the individual person's level but, of course, their contribution to the average level of deprivation in the population as a whole is correspondingly smaller. Factors $w_k^b$ in (9) are computed as follows:

$$w_k^b \propto \left( \frac{1}{1+\sum_{k'=1}^{K} \rho_{k,k'} \mid \rho_{k,k'} < \rho_H} \right) \cdot \left( \frac{1}{\sum_{k'=1}^{K} \rho_{k,k'} \mid \rho_{k,k'} \geq \rho_H} \right), \quad (11)$$

where $\rho_{k,k'}$ is the correlation between the two indicators. In the first factor of the equation, the sum is taken over all indicators whose correlation with the variable k is less than a certain value $\rho_H$ (determined, for instance, by dividing the ordered set of correlation values at the point of the largest gap). The sum in the second term always includes the case k' = k, since that correlation coefficient is 1.0. The motivation for this model is that (i) $w_k^b$ is not affected by the introduction of variables entirely uncorrelated with k; (ii) it is only marginally affected by small correlations; but (iii) is reduced proportionately to the number of highly correlated variables present.[8] Factors $w_k^b$ can be calculated either separately for any class of

[8] In practice we have mostly found that, on the basis of the 'largest gap' criterion, the second factor involves only the variable itself (i.e., is reduced to 1), so that the weight of a variable is simply inverse of the average of correlations with all the variables (including the variable concerned itself).

items with respect to which we want to measure deprivation, or for all the items together when measuring overall non-monetary deprivation. Finally, the scaling of the weights can be arbitrary, though scaling them to sum to 1.0 may be convenient.

### 2.5 Income poverty and non-monetary deprivation in combination: Manifest and Latent deprivation

In the previous sections we have considered poverty as a fuzzy state and defined measures of its degree in different dimensions, namely: monetary, overall non-monetary and regarding particular aspects of life. In multidimensional analysis it is of interest to know the extent to which deprivation in different dimensions tends to overlap for individuals.[9] Such analyses require the specification of rules for the manipulation of fuzzy sets, such as defining set complements, intersections, unions and aggregations.

As a concrete example let us consider deprivation in two dimensions: income deprivation and overall non-monetary deprivation that we denote by *m* and *s* respectively, each of them being characterised by two opposite fuzzy states labelled as 0 (non-deprived) and 1 (deprived), that correspond to a pair of fuzzy sets forming a fuzzy partition.

According to our previous definitions, any individual i belongs *to a certain degree* to the two cross-sectional sets and to their complements, as illustrated in Table 2a.

Note that since fuzzy sets 0 and 1 are complementary, having defined the degree of membership in 1 as $FM_i$ or $FS_i$, it is straightforward (and necessary) to calculate the membership in set 0 as $(1 - FM_i)$ or $(1-FS_i)$, respectively. In the conventional approach, a joint analysis of monetary and non-monetary deprivation (both seen as dichotomous characteristics) is carried out by classifying any individual in one (and only one) of the four sets representing the intersections $m \cap s$ ($m$ = 0,1 ; $s$ = 0,1). Table 2b

[9] Similarly, in longitudinal analysis it would be of interest to know the extent to which the state of poverty or deprivation persists over time for the person concerned.

exemplifies the case of an individual affected by monetary deprivation only[10].

**Table 2a. Membership functions of an individual in the 4 intersection sets**

| Poverty dimension | State | | Individual degree of membership |
|---|---|---|---|
| *m* | non-deprived | 0 | $1 - FM_i$ |
| | deprived | 1 | $FM_i$ |
| *s* | non-deprived | 0 | $1 - FS_i$ |
| | deprived | 1 | $FS_i$ |

**Table 2b. Situation of a hypothetic individual**

| Monetary deprivation (*m*) | Non monetary deprivation (*s*) | | |
|---|---|---|---|
| | poverty status | non-poor (0) | poor (1) |
| | non-poor (0) | 0 | 0 |
| | Poor (1) | 1 | 0 |

After classifying all the *n* surveyed individuals, the collective situation is analysed and quantified on the basis of their joint distribution in the four sets represented in Table 2c.

Differently, when we treat the two types of deprivation *m* and *s* in fuzzy terms, any individual belongs to each of the four intersections with degree of membership varying between 0 and 1.

Moreover, since these four sets form a fuzzy partition, their respective degrees of membership must sum to 1.

[10] I.e., an individual classified in the set $\{(m=1)\cap(s=0)\}$ with degree of membership equal to 1.

More precisely, denoting by $\mu_{ims}$ the degree of membership in $m \cap s$ (m = 0,1 ; s = 0,1) of individual i, the marginal constraints pointed out in Table 4 must be satisfied.

**Table 2c. Joint distribution of the *n* surveyed individuals according to both types of deprivations**

| | | Non monetary deprivation (*s*) | | |
|---|---|---|---|---|
| Monetary deprivation (*m*) | poverty status | non-poor (0) | poor (1) | total |
| | non-poor (0) | $n_{00}$ | $n_{01}$ | $n_{0.}$ |
| | poor (1) | $n_{10}$ | $n_{11}$ | $n_{1.}$ |
| | total | $n_{.0}$ | $n_{.1}$ | $n$ |

**Table 3. Situation of a generic individual i seen in fuzzy terms**

| | | Non-monetary deprivation (*s*) | | |
|---|---|---|---|---|
| Monetary deprivation (*m*) | poverty status | non-poor (0) | poor (1) | total |
| | non-poor (0) | $\mu_{i00}$ | $\mu_{i01}$ | $1 - FM_i$ |
| | Poor (1) | $\mu_{i10}$ | $\mu_{i11}$ | $FM_i$ |
| | total | $1 - FS_i$ | $FS_i$ | 1 |

In practice, $\mu_{ims}$ represents a measure of the extent to which the individual is affected by the particular combination of states (*m*,*s*).

The specification of the fuzzy intersection $\mu_{ims}$ that appears to be the most reasonable for our particular application is the one illustrated in Table 4.

**Table 4. Joint measures of deprivation**

| | Non-monetary deprivation (*s*) | | | |
|---|---|---|---|---|
| | poverty status | non-poor (0) | poor (1) | total |
| Monetary deprivation (*m*) | non-poor (0) | $\min(1-FM_i, 1-FS_i)$ $=1-\max(FM_i, FS_i)$ | $\max(0, FS_i-FM_i)$ | $1 - FM_I$ |
| | Poor (1) | $\max(0, FM_i-FS_i)$ | $\min(FM_i, FS_i)$ | $FM_i$ |
| | total | $1 - FS_i$ | $FS_i$ | 1 |

Such a specification results from the combination of the so called "standard" and "bounded" rules for fuzzy intersection[11] and the justification of this choice can be summarised as follows[12]:

(i) it satisfies the marginal constraints;

(ii) it takes into account the positive correlation that exists between the two types of deprivation (monetary and non-monetary);

(iii) it reproduces the crisp set intersection (the conventional situation exemplified in Tables 2b and 2c above) when the fuzzy memberships, being in the whole range [0,1], are reduced to a {0,1} dichotomy.

In practice, while property (iii) is a sort of pre-requisite for any fuzzy set operation, property (i) would be satisfied also by the so called "algebraic" intersection, which is simply the product of the two corresponding marginal memberships. However, we rejected the algebraic rule because it implies the absence of correlation between the two types of deprivation.

The two measures – $FM_i$ the propensity to income poverty, and $FS_i$ the overall non-monetary deprivation propensity - may be combined to construct composite measures which indicate the extent to which the two aspects of income poverty and non-monetary deprivation overlap for the individual concerned. These measures are as follows.

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[11] Cf. Klir and Yuan (1995).

$M_i$ Manifest deprivation,

representing the propensity to both income poverty and non-monetary deprivation simultaneously. One may think of this as the 'more intense' degree of deprivation.

$L_i$ Latent deprivation,

representing the individual being subject to at least one of the two, income poverty and/or non-monetary deprivation; one may think of this as the 'less intense' degree of deprivation.

Once the propensities to income poverty ($FM_i$) and non-monetary deprivation ($FS_i$) have been defined at the individual level (i), the corresponding combined measures are obtained as follows:

$$M_i = \min(FM_i, FS_i) \tag{12}$$

$$L_i = \max(FM_i, FS_i) \tag{13}$$

The Manifest deprivation propensity of individual i is the intersection (the smaller) of the two measures $FM_i$ and $FS_i$. Similarly, the Latent deprivation propensity of individual i is the union (the larger) of the two measures $FM_i$ and $FS_i$.

It can be shown that the estimates provided by the Standard rule used in (12) and (13) are maximal for the intersections and minimal for the union, so that we have a *maximal* estimate for Manifest deprivation, and a *minimal* for Latent deprivation. We argue that on substantive grounds, this is a reasonable (indeed desirable) choice for intersections of 'similar' states.

## 3. Longitudinal measures

The combination of the "standard" and "bounded" operations introduced in the previous section are here utilised when we intend to calculate the joint membership function to two (or more) consecutive time periods.

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[12] For more details see Betti *et al.* (2005).

### 3.1 Longitudinal measures over two time periods

When analysing the situation over two periods we are interested to joint membership function to four fuzzy states: state of poor at time 1 and poor at time 2, state of poor at time 1 and non poor at time 2, state of non poor at time 1 and poor at time 2 and, finally, state of non poor at time 1 and non poor at time 2. Those joint states can identify (as well as in the traditional approach) the concepts of persistent poverty, exiting from poverty, entering into poverty, etc. As described in Table 5.

**Table 5. Longitudinal measures of interest over two time periods**

| | Measure | Membership function | Description |
|---|---|---|---|
| 1 | Never in poverty | $\bar{\mu}_1 \cap \bar{\mu}_2 = 1 - \max(\bar{\mu}_1, \bar{\mu}_2)$ | Poverty at *neither* of the two years |
| 2 | Persistent in poverty | $\mu_1 \cap \mu_2 = \min(\mu_1, \mu_2)$ | Poverty at *both* of the years |
| 3 | Exiting from poverty | $\mu_1 \cap \bar{\mu}_2 = \max(0, \mu_1 - \mu_2)$ | Poverty at time 1, but non-poverty at time 2 |
| 4 | Entering into poverty | $\bar{\mu}_1 \cap \mu_2 = \max(0, \mu_2 - \mu_1)$ | Non-poverty at time 1, but poverty at time 2 |
| 5 | Ever in poverty | $\mu_1 \cup \mu_2 = \max(\mu_1, \mu_2)$ | Poverty at *at least one* of the two years |

### 3.1 Longitudinal measures over more than two time periods

Analysis of the persistence of poverty over more than two time periods requires also the specification of j.m.f.'s of the type: $I_T = \mu_1 \cap \mu_2 ........\cap \mu_T$ and $U_T = \mu_1 \cup \mu_2 ........\cup \mu_T$, where the first expression is the intersection of a series of T cross-sectional m.f.'s for any individual unit, and the second expression is their union.

Since all sets $\mu_1,......,\mu_T$ are of the <u>same type</u> (all being propensities to "poverty" rather than to "non-poverty"), the "standard" operations apply:

$$I_T = \min(\mu_1, \mu_2, ......, \mu_t, .....\mu_T) \quad (14)$$

$$U_T = \max(\mu_1, \mu_2, ......, \mu_t, .....\mu_T). \quad (15)$$

$I_T$ represents the individual's propensity to be poor at all T periods.

$U_T$ is the propensity to be poor at *at least one* of the T periods; the propensity to be non-poor over all T periods is its complement $\overline{U}_T = 1 - U_T$.

The same result is obtained by considering intersection of non-poor sets:

$$\overline{I}_T = \min(\overline{\mu}_1, \overline{\mu}_2, ......, \overline{\mu}_t, .....\overline{\mu}_T) = 1 - \max(\mu_1, \mu_2, ......., \mu_T) = 1 - U_T.$$

The propensity of experiencing poverty over any specific sequence of t out of T years is given by the minimum value of cross-sectional propensities $\mu$ over those particular years, representing the intersection between the t similar states.

**Any time poverty:**

membership function of the set "poor for at least one year" = $U_T$

**Continuous poverty:**

membership function of the set "poor for all the T years" = $I_T$.

**Persistent poverty:**

We may define persistent poverty as the propensity to be poor over at least a majority of the T years, i.e. over at least t years, with t=int(T/2)+1, the smallest integer strictly larger than (T/2).

For instance, for a T= 4 or 5 year period, 'persistent' would refer to poverty for at least 3 years; for T =6 or 7, it would refer to poverty for at least 4 years, etc. The required propensity to persistent poverty is the [int(T/2)+1]$^{th}$ largest value in the sequence ($\mu_1$,……$\mu_T$).

### 3.3 Rates of exit and re-entry

Given the state of poverty at time 1, and also at a later time (t-1), what is the proportion exiting from poverty at time t=2, 3, …?

Given the state of poverty at time 1, but of non-poverty at a later time (t-1), what is the proportion which has re-entered poverty at time t=3, 4, ….?

In conventional analysis, the above rates are computed simply from the count of persons in various states:

For exit rate, the numerator is the count of persons poor at times 1 and (t-1), but non-poor at time t; the denominator is the count of all persons who are poor at times 1 and t-1 (and are present in the sample at time t).
For re-entry rate, the numerator is the count of persons poor at time 1, non-poor at time (t-1), but poor again at time t. The denominator is the count of persons who are poor at time 1 and non-poor at time (t-1) (and are present in the sample at time t).
The construction of these measures using fuzzy m.f.'s is also straightforward. With $\mu_t$ as a person's propensity to poverty at time t, the person's contribution of these rates is as follows.
Exit rate:

Numerator $(\mu_1 \cap \mu_{t-1}) \cap \overline{\mu}_t = \max[0, \min(\mu_1, \mu_{t-1}) - \mu_t]$

Denominator $(\mu_1 \cap \mu_{t-1}) = \min(\mu_1, \mu_{t-1})$

*Re-entry rate:*

Numerator $\mu_1 \cap \overline{\mu}_{t-1} \cap \mu_t = (\mu_1 \cap \mu_t) \cap \overline{\mu}_{t-1} = \max[0, \min(\mu_1, \mu_t) - \mu_{t-1}]$

Denominator $\mu_1 \cap \overline{\mu}_{t-1} = \max[0, \mu_1 - \mu_{t-1}]$

## 4. Empirical analysis

We have calculated the traditional poverty index $H_i$ and the Fuzzy Monetary index $FM_i$ for individuals in the Italian European Community Household Panel survey from 1994 to 2001 (income reference years from 1993 to 2000).
Table 6, first panel, shows the conventional poverty rates H for Italy and her five Macro-regions. With the household's equivalised income ascribed to each of its members, persons with equivalised income below 60% of the national median have been classified as poor. This is done for each ECHP wave separately. The results are also shown averaged over the eight waves so as to gain sampling precision and identify more clearly the overall patterns across Macro-regions. Such consolidation is also necessary in this paper for reasons of space, and is required even when it

would have been more illuminating to present results for individual waves.[13]

This suggests the alternative of obtaining a single, more robust value of $\alpha$ *by pooling together data from all waves being analysed*. This benchmarks the fuzzy poverty rate to be identical to the conventional rate for *the group of waves as a whole for all-Italy*. This pooled approach has the advantage that only a single parameter has to be estimated, which can therefore be done more reliably.[14] Levels of Macro-regional median incomes are also shown in Table 6 for reference. For the set of Macro-regions considered, there is generally a *negative* relationship between the income level and the relative poverty rate. This results in part from differences in regional mean incomes (since a common national poverty line has been used), but it also reflects the considerable differences in the levels of inequality within regions. As can be seen from comparing individual cells in the two panels of Table 6, the fuzzy and conventional poverty rates are quite similar to each other. In fact, the ratio ($FM_W$ / $H_W$) is quite stable across waves within each Macro-region. The ratio (FM / H) is also similar, though less uniform, across Macro-regions. It also tends to decrease a little with increasing H, meaning that the fuzzy measures show slightly smaller differentials in the Macro-regional poverty rates. Note that the two measures FM and H are constructed, by definition, to be identical at the all-Italy level averaged over the 8 waves.[15]

[13] It is important to note that all results presented after Table 6 are based on detailed wave-specific computations, but have been averaged over waves in the presentations for reasons noted above.

[14] Note that in order to define $\alpha$, the quantities $(1-L_{(M),i})$ must still be defined separately for each survey wave. It is only after that that the data are pooled across waves to determine $\alpha$ iteratively.

[15] This is because this constraint was used in determining the value of the parameter $\alpha$ as described above.

## Table 6. Conventional and fuzzy cross-sectional measures of the income poverty rate

| ECHP wave | w1 | w2 | w3 | w4 | w5 | w6 | w7 | w8 | 8 ECHP waves | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Head count ratio (H)** | | | | | | | | | **H** | **EqInc** |
| Italy | 20,4 | 20,4 | 20,1 | 19,7 | 18,0 | 18,0 | 18,5 | 19,3 | **19,3** | 100 |
| NORDOVEST | 10,2 | 10,4 | 9,0 | 9,2 | 9,1 | 6,8 | 7,0 | 7,2 | **9,8** | 121 |
| NORDEST | 12,6 | 10,8 | 7,8 | 7,5 | 6,5 | 6,6 | 5,8 | 5,8 | **7,9** | 120 |
| CENTRO | 14,5 | 13,2 | 14,5 | 15,1 | 12,8 | 11,9 | 13,6 | 16,1 | **14,0** | 103 |
| SUD | 33,2 | 34,2 | 34,2 | 33,5 | 30,1 | 33,0 | 32,2 | 33,3 | **33,0** | 76 |
| ISOLE | 40,0 | 37,2 | 40,5 | 37,8 | 36,3 | 37,7 | 40,6 | 40,8 | **38,8** | 73 |
| ECHP wave | w1 | w2 | w3 | w4 | w5 | w6 | w7 | w8 | 8 ECHP waves | |
| **Fuzzy monetary (FM) poverty rate** | | | | | | | | | **FM** | **FM/H** |
| Italy | 19,4 | 19,4 | 19,3 | 19,3 | 19,3 | 19,2 | 19,2 | 19,2 | **19,3** | 1,00 |
| NORDOVEST | 11,5 | 11,0 | 10,4 | 10,3 | 10,9 | 9,1 | 9,1 | 9,5 | **10,2** | 1,05 |
| NORDEST | 13,0 | 11,4 | 9,9 | 10,0 | 8,9 | 9,2 | 8,7 | 8,7 | **10,0** | 1,26 |
| CENTRO | 15,4 | 14,7 | 15,2 | 16,1 | 16,4 | 15,8 | 16,4 | 17,2 | **15,9** | 1,14 |
| SUD | 29,3 | 30,2 | 30,2 | 30,8 | 30,0 | 31,8 | 30,8 | 30,2 | **30,4** | 0,92 |
| ISOLE | 34,2 | 32,9 | 35,2 | 32,9 | 33,9 | 35,0 | 36,3 | 35,9 | **34,5** | 0,89 |

| | |
|---|---|
| HCR | head-count ratio (conventional monetary poverty rate) |
| FM | fuzzy measure of monetary poverty rate ('Fuzzy Monetary') |
| EqInc | mean equivalised household income (relative to IT mean=100) |
| waves | w1: survey year 1994 (income reference year 1993). w8: survey year 2001. |

Table 7 compares fuzzy measures of income poverty and of non-monetary deprivation across Italian Macro-regions. For reasons noted, the two measures have been averaged over 8 waves, and are scaled to be identical to each other at all-Italy level. Macro-regions with low levels of monetary poverty indicate a higher level of non-monetary deprivation compared to their level of monetary poverty. Overall, there is a notable negative correlation between the level of income poverty (FM) and the ratio (FS/FM), though in actual values the two measures (FM, FS) are quite similar and equally relative.

Returning to Table 7, the last column of the table shows Manifest deprivation index as percentage of Latent deprivation index: it can be interpreted as an index of the degree of overlap (or intersection), at the level of individual persons, between income poverty and non-monetary deprivation.

**Table 7. Fuzzy measures of deprivation: monetary, non-monetary, and the two forms in combination**

| | Fuzzy deprivation rates | | | | | Ratios | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | FS/ | Manifest/ | Latent/ | Manifest/ |
| | FM | FS | Manifest | Latent | Mean | FM | Mean | Mean | Latent |
| Italy | 19,3 | 19,3 | 9,3 | 29,3 | 19,3 | 1,00 | 0,48 | 1,52 | 0,32 |
| NORDOVEST | 10,2 | 15,0 | 4,0 | 21,2 | 12,6 | 1,47 | 0,32 | 1,68 | 0,19 |
| NORDEST | 10,0 | 11,1 | 3,6 | 17,5 | 10,5 | 1,11 | 0,34 | 1,66 | 0,20 |
| CENTRO | 15,9 | 16,5 | 6,7 | 25,7 | 16,2 | 1,04 | 0,41 | 1,59 | 0,26 |
| SUD | 30,4 | 27,2 | 16,4 | 41,2 | 28,8 | 0,90 | 0,57 | 1,43 | 0,40 |
| ISOLE | 34,5 | 28,5 | 18,4 | 44,6 | 31,5 | 0,82 | 0,58 | 1,42 | 0,41 |

FM fuzzy measure of monetary poverty rate ('fuzzy monetary')
FS fuzzy measure of overall non-monetary deprivation rate ('fuzzy supplementary')
Manifest propensity to both FM and FS deprivation
Latent propensity to either form of deprivation (FM and/or FS)
Mean mean of (FM,FS) = mean of (Manifest, Latent)
Note: Figures show simple averages of cross-sectional results over 8 ECHP waves

In theory, this ratio varies from 0 to 1. When there is no overlap (i.e., when the subpopulation subject to income poverty is entirely different from the subpopulation subject to non-monetary deprivation), Manifest deprivation rate and hence the above mentioned ratio equals 0. When there is complete overlap (i.e., when exactly the same subpopulation is subject both to income poverty and to non-monetary deprivation), the Manifest and Latent deprivation rates are the same and hence the above mentioned ratio equals 1.

It is important to highlight that there is a *higher* degree of overlap between income poverty and non-monetary deprivation at the level of individual persons in *poorer* Macro-regions, and a lower degree of overlap in richer Macro-regions of Italy. This leads to the conclusion that the adoption of a multi-dimensional approach is particularly important when analysing richer regions (or for that matter, countries in an international study), where different dimensions have less overlap. Therefore in this cases the adoption of a supplementary indicator as a complement to the monetary one is justified, because it has an added value. On the other hand, because of the higher degree of overlap in poorer (and less equal) regions, the overall deprivation is more intense for the subpopulations involved, which is also important. All this underlines the need to supplement monetary indicators by multi-dimensional measures.

### Table 8. Longitudinal measures: traditional vs. fuzzy approach

Distribution of the popualation according to the number of years in poverty during past 8 years

| Conventional monetary poverty measure<br>Macro-Region | zero | 1 yr | 2 yr | 3 yr | 4 yr | 5 yr | 6 yr | 7 yr | 8 yr | mean years during the 8 yr period |
|---|---|---|---|---|---|---|---|---|---|---|
| NORDOVEST | 73,53 | 11,35 | 6,15 | 3,55 | 2,61 | 1,26 | 0,05 | 1,10 | 0,40 | 0,62 |
| NORDEST | 73,51 | 12,04 | 6,42 | 3,82 | 0,89 | 1,53 | 0,46 | 0,79 | 0,53 | 0,60 |
| CENTRO | 58,87 | 14,99 | 8,92 | 5,25 | 2,86 | 2,54 | 4,56 | 0,72 | 1,28 | 1,15 |
| SUD | 40,04 | 13,23 | 7,99 | 8,28 | 5,00 | 5,18 | 7,20 | 7,36 | 5,72 | 2,40 |
| ISOLE | 32,86 | 12,35 | 7,36 | 8,97 | 6,69 | 5,94 | 5,58 | 10,75 | 9,48 | 2,95 |
| ITALY | 57,42 | 12,80 | 7,36 | 5,76 | 3,43 | 3,10 | 3,46 | 3,67 | 3,00 | 1,44 |

| Fuzzy monetary poverty measure<br>Macro-Region | zero | 1 yr | 2 yr | 3 yr | 4 yr | 5 yr | 6 yr | 7 yr | 8 yr | mean years during the 8 yr period |
|---|---|---|---|---|---|---|---|---|---|---|
| NORDOVEST | 74,32 | 9,63 | 5,02 | 2,78 | 2,45 | 1,68 | 1,13 | 1,31 | 1,67 | 0,76 |
| NORDEST | 75,62 | 8,61 | 4,53 | 3,05 | 1,92 | 1,52 | 1,27 | 1,22 | 2,24 | 0,76 |
| CENTRO | 63,92 | 9,91 | 6,13 | 4,27 | 3,20 | 2,93 | 2,75 | 2,88 | 3,99 | 1,31 |
| SUD | 47,99 | 10,73 | 6,32 | 5,28 | 4,89 | 4,41 | 4,91 | 6,08 | 9,39 | 2,28 |
| ISOLE | 43,06 | 10,65 | 6,43 | 5,63 | 5,16 | 4,47 | 4,85 | 7,07 | 12,67 | 2,63 |
| ITALY | 62,21 | 9,90 | 5,65 | 4,09 | 3,43 | 2,91 | 2,86 | 3,48 | 5,47 | 1,47 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| NORDOVEST | 0,99 | 1,18 | 1,23 | 1,28 | 1,07 | 0,75 | 0,04 | 0,84 | 0,24 | 0,82 |
| NORDEST | 0,97 | 1,40 | 1,42 | 1,25 | 0,46 | 1,01 | 0,36 | 0,65 | 0,24 | 0,79 |
| CENTRO | 0,92 | 1,51 | 1,45 | 1,23 | 0,89 | 0,87 | 1,66 | 0,25 | 0,32 | 0,88 |
| SUD | 0,83 | 1,23 | 1,26 | 1,57 | 1,02 | 1,17 | 1,47 | 1,21 | 0,61 | 1,05 |
| ISOLE | 0,76 | 1,16 | 1,14 | 1,59 | 1,30 | 1,33 | 1,15 | 1,52 | 0,75 | 1,12 |
| ITALY | 0,92 | 1,29 | 1,30 | 1,41 | 1,00 | 1,06 | 1,21 | 1,05 | 0,55 | 0,98 |

Table 8 reports some longitudinal measures according to both the traditional and the fuzzy monetary approach. In particular are reported the distribution of years spent in poverty (traditional approach) and the average membership function to joint states of zero to eight poor states.
From the Table is evident how in the traditional approach the transitory poverty is overestimated (from 1 to 4 years in poverty) and the permanent poverty is underestimated (from 5 to 8 years in poverty).

## 5. Policy implications

First of all, from the empirical analysis conducted in Section 4, and also from other previous experiences on European countries (Eurostat, 2003; Betti and Verma, 2002) it is important to highlight how the multidimensional approach to poverty measurement is important to complement the monetary approach (traditional or fuzzy) when analysing living condition in developed countries. Smaller is the proportion of poor people in a community, less is the degree of overlapping between monetary and non deprivation, and therefore less powerful are all those policies that are simply based on monetary transfers to "monetary" poor individuals or families. Moreover a deep analysis based on several dimensions of deprivation can lead the policy maker to individuate the right "spheres" of the life-style where to orient the anti-poverty measures (housing conditions, environmental problems, etc…).

Secondly, a fuzzy monetary approach clearly is able to avoid the overestimation of the transitory poverty that characterised the longitudinal traditional approach. The real quantification of the transitory and persistent poverty (and its disaggregation over selected "profiles") can instruct policy makers in adopting measures that are able to combat the structural causes of persistent poverty.

## 6. Concluding remarks

The aim of this paper has been to develop and refine the strand of research which started with the contributions of Cerioli and Zani (1990) and then has been followed by a number of applications and developments. Methodologically, the implementation of this approach has developed in two directions, with somewhat different emphasis despite their common orientation and framework: one emphasising more the multidimensionality and the other more the longitudinal aspects of poverty analysis. We have tried to bring together various strands of development on the subject. Specifically, we address in this paper the *additional* factors which the introduction of fuzzy approach, as distinct from the conventional approach, brings into the analysis of poverty and deprivation. Choices have to be made in relation to these, involving at least two aspects:

- Choice of 'membership functions', that is, quantitative specification of the propensity to poverty and deprivation of each person in the population, given the level and distribution of income and other measures of living standards.

- Choice of 'rules' for manipulation of the resulting fuzzy sets, specifically the rules defining complements, intersections, union and aggregation of the sets.

A major objective has been to clarify how both these choices must meet some basic logical and substantive requirements to be meaningful. We believe that, hitherto, the rules of fuzzy set operations in the context of poverty analysis have not been well or widely understood.

The focus in this paper has been on the multidimensional aspects of deprivation. Persistence and movement over time is an equally important aspect of the intensity of deprivation, requiring longitudinal study at the micro level and in the aggregate; in the paper we have also presented a methodology for estimating fuzzy longitudinal poverty measures.

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