International Conference

The many dimensions of poverty

Brasilia, Brazil – 29-31 August 2005
Carlton Hotel

Statistical Tests for Multidimensional Poverty Analysis

Conference paper

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This text is on embargo until 29 August

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Statistical Tests For Multidimensional Poverty Analysis

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June 2005

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1. Introduction

At least nominally if not in fact, poverty reduction has been the espoused policy target of nations and global institutions in recent years, putting demands upon "Chiffrefilic" economists to quantify it and measure its progress. Like many things in life, it is hard to define, but you know it when you see it and typically, the more instruments available to describe it, the better it is described! Indeed Sen's arguments (see Sen (1992)) - that welfare and inequality, when measured in terms of functioning's and capabilities, is intrinsically a many dimensioned thing - are equally pertinent for poverty measurement. When confined to the single variable paradigm the measurement and testing of poverty states has prompted many questions: "What variable should be employed (income or consumption)?", "What should the poverty cut-off point be?", "How should the variable be transformed (incidence, depth or intensity formulations)?", "Should we use permanent or transitory concepts?". These issues are both diminished and compounded in magnitude when we move to a multi-dimensional paradigm: to some extent variable choice becomes less of a problem (if in doubt include as much as possible) but how the combination of the factors in defining what would be a poverty boundary and the extent of poverty it delimits presents a whole new set of questions.

Many of these problems have been resolved or avoided by the great contributions of Atkinson (1987) and by extension Duclos et. al. (2001). Their results can broadly be summarized as follows. If a stochastic dominance relationship of a given order can be established between two characteristic distributions over a region that includes all of the relevant values of the characteristics, then definitive statements about the progress of poverty can be made for any poverty indicator within a known class and for any poverty boundary within the region. To the practical chiffrefile these results are liberating, they remove the focus of debate from what sort of poverty measure and what poverty boundary should be employed to whether poverty (however it is measured) has actually increased or diminished in some probabilistic sense. In any discussion of tests for multivariate poverty they must be the first port of call. However they are not always useful, the orderings are only partial (that is, sometimes orderings cannot be established which is not to say that the state of poverty has not changed, merely that the change, if any, cannot be identified by the technique) and they do not answer the relative magnitude "by how much and is it significant?" type of question policy makers frequently pose. Furthermore in the multivariate case stochastic dominance techniques soon run into data constraints commonly referred to in the non-parametric statistics literature as the curse of dimensionality (Anderson (2004)).

To answer the "by how much?" question and overcome the degrees of freedom problem it is incumbent upon us to define a poverty frontier and some sort of characteristic weighting procedure which yields a univariate poverty index and figure out its statistical distribution. The problem of defining a poverty frontier is what should the "tradeoffs" be on the frontier between what we shall refer to as goods[1]? One approach has been to define for each good "i" consumed by agent "j" x_{ij} (where in particular i=1,..,G), a poverty cutoff level z_i for the i'th good, then two extremes exist, either an agent is in poverty if $x_{ij} < z_i$ for any i (The set union rule) or $x_{ij} < z_i$ for all i (The set intersection rule) (see for example Deustch and Silber (2005) and Duclos et. al. (2002)). These are indeed extremes and as such they have peculiar consequences when the number of agent characteristics considered increases. For example, suppose for convenience the poverty cutoff with respect to a good is any consumption level less than half the population median of that good, and suppose further that all goods are independently uniformly distributed throughout the population. For G=1 the poverty rate = ¼ for both union and intersection rules, for 2 goods the poverty rate would be 7/16 for the union rule and 1/16 for the intersection rule, for 3 goods the corresponding rates would be 37/64 and 1/64 respectively and so on. In effect, as the number of goods tends to infinity the poverty rate goes to zero by the intersection rule and to 1, or the whole population, by the union rule and it does so pretty rapidly in the example presented. Median cut-offs for two goods are illustrated in figure 4 below in section 3. For the intersection rule all of the points in the dashed rectangle would be considered agents in poverty, for the union rule all of the agents outside of the upper solid line quadrant would be considered in poverty.

[1] One of the primary motivations for a multivariate approach to poverty analysis are the arguments for characterizing welfare in terms of functionings and capabilities (see annexe 7 in Sen (1997) for example), without loss of generality these will simply referred to as goods.

Intuitively the union rule treats goods as completely non-substitutable in the poverty sense, deprivation in one of them is deprivation in all of them whereas the intersection rule treats them as perfectly substitutable in the poverty sense, poverty deprivation only occurs when there is deprivation in all goods. Of course the reality is that there will be some trade-offs on the boundary with respect to at least some of the goods, agents may well find an increase in social deprivation in exchange for a reduction in material deprivation quite acceptable at the margin. In order to obtain a poverty boundary that makes some intuitive sense, the natural route for an economist to take is to posit some sort of agent welfare function $W(x_i)$ whose arguments, the vector x_i, are the goods (functionings and capabilities) of interest with respect to agent "i" and then to define some welfare level W* below which agent "j" is deemed poor when $W(x_j) < W^*$. Herein lies a problem in that W is itself fundamentally unobservable and can only be estimated up to a factor of proportionality. Furthermore for a specification of W( ) that is integrable, that is to say for which the parameters of W( ) can be recovered from agent characteristic demand functions, details of the constraints (i.e. prices and incomes) are required which, in the present context, are almost never available.

The solution presented here is to employ relative concepts of welfare and hence poverty which have recently met with some popularity in terms of employing poverty cut-offs which are proportionate to the population median. Actually the idea is not so new, Adam Smith (1776) can be interpreted to have had a similar view viz: "..By necessaries I understand, not only the commodities which are indispensably necessary for the support of life, but whatever the custom of the country renders it indecent for creditable people, even the lowest order, to be without." Similarly Ferguson (1767) states "The necessary of life is a vague and relative term: it is one thing in the opinion of the savage; another in that of the polished citizen: it has a reference to the fancy and to the habits of living". As will be seen such a notion liberates us from the need to specify and estimate agent preferences in order to obtain a poverty boundary and at the same time provides some other insights that would be of interest to empirical welfarists. The approach is based upon the notion of distance between elements in multidimensional space and the concept of the lower convex hull of those elements. The dimensions correspond to goods or capabilities or functionings which contribute positively to an agents well-being and the elements or points in the multidimernsional space correspond to the status of the agents. The novelty in the approach is the use of the lower convex hull which corresponds to the set of “poorest” agents, what will be referred to (and interpreted) as the “Rawlsian Boundary”, and to which the poverty frontier will be related.

In the following section 2 briefly outlines some alternative approaches to multivariate poverty analysis. Aspects of the distance approach together with the “Rawlsian Frontier” and the associated Poverty Frontiers are discussed in section 3. The results from implementing the technique are reported in section 4 and section 5 concludes.

2. Some Alternative Approaches to Multivariate Poverty Rankings.

D’Ambrozio, Deutsch and Silber (2004) and Deutsch and Silber (2005) identified four approaches the Fuzzy Set, Information Theory, Axiomatic and Distance Function approaches to multivariate poverty measurement in a study of how the different approaches exhibited consistency in identifying the same agents in the poverty group.

In the Fuzzy Set approach (Cheli and Lemmi (1995)) the poverty boundary becomes a space with an associated probability distribution assigning members to the poverty group or otherwise. There are two possible approaches: a) The Totally Fuzzy Approach where the boundary space has pre-defined upper and lower bounds and b) The Relative Totally Fuzzy Approach wherein the boundary space is the complete factor set, the latter avoids the need to specify a poverty boundary.

Letting $\xi_{j(m)}$, m=1,..,s be the set of rank ordered values that the variable ξ_j (a measure of the state of deprivation with respect to the j’th factor) can take on, where the ordering is in terms of increasing risk of poverty (i.e.$\xi_{j(1)}$ implies lowest risk and $\xi_{j(s)}$ implies highest) and let its cumulative marginal density be F_j . Then μ_{ji}, the degree of poverty associated with the j’th factor for the i’th individual, is defined recursively as:

$$\mu_{ji} = 0 \quad if \ \xi_{ji} = \xi_{j(1)}$$

$$= F_j(\xi_{j(m-1)}) + \frac{F_j(\xi_{j(m)}) - F_j(\xi_{j(m-1)})}{1 - F_j(\xi_{j(1)})} \quad for \ m > 1$$

Aggregation of the poverty indices across the poverty factors is a weighted sum of the individual poverty factors so that:

$$P = (\frac{1}{n})\sum_{i=1}^{n}\sum_{j=1}^{k} w_j \mu_{ji}$$

Where the weights are of the form:

$$w_j = \frac{\ln(n^{-1}\sum_{i=1}^{n}\mu_{ji})}{\sum_{j=1}^{k}\ln(n^{-1}\sum_{i=1}^{n}\mu_{ji})}$$

Here the poverty space is potentially the whole characteristic space and the weighting function is the proportions of the log marginal cumulative densities corresponding to the welfare aggregator.

The Information Theory approach (Maassoumi (1986)) is really a technique for assessing inequality. Given an optimal aggregator (or agent utility) function of the form:

$$S_i = (\sum_{j=1}^{m}\delta_j X_{ij}^{-\gamma})^{-\frac{1}{\gamma}}; \quad \gamma \neq 0,1$$

$$= \prod_{j=1}^{m} X_{ij}^{\delta_j}; \quad \gamma = 0$$

$$= \sum_{j=1}^{m}\delta_j X_{ij}; \quad \gamma = -1.$$

Inequality is studied via a Generalized entropy family defined as:

$$D(S,X,\theta,\beta)=\sum_{j=1}^{m}\beta_j\left(\frac{\sum_{i=1}^{n}S_i\left(\left(\frac{S_i}{X_{ij}}\right)^{\theta}-1\right)}{\theta(\theta+1)}\right)$$

Again specification of an aggregator function is necessary in order to make the procedure operational and extension to the analysis of poverty requires the definition of a poverty boundary in terms of the distribution of S.

In the Axiomatic approach Tsui (1995) provides Relative and Absolute multidimensional measures which are generalizations of the Atkinson(1970) - Kolm (1966) - Sen (1997) "Ethically Based" family and followed with the corresponding multivariate poverty indices in a similar vein to Chakravarty, Mukherjee and Ranade (1998). These are multivariate generalizations of the Foster, Greer, Thorbecke (1984) indices the one employed in this paper was the second order ("Poverty Depth") index of the form

$$P(X,z)=\left(\frac{1}{n}\right)\sum_{j=1}^{k}\sum_{i=1,i\in S_j}^{n}\frac{1}{k}\left(1-\frac{X_{ij}}{z_j}\right)$$

Where z_i i=1,..,k corresponds to the level of basic need for factor i (set at half the mean value for the respective indicator), note equal weight was given to all indicators. In the D'Ambrozio et.al.(2004) analysis an individual "i" was considered poor when:

$$\sum_{j=1}^{k}\frac{1}{k}\left(1-\frac{X_{ij}}{z_j}\right)$$

was greater than the 75th percentile in the sample.

Here a basic need (poverty bound) is defined for each characteristic, there are no trade-offs between the basic needs on the boundary, and if an agent was in the top 25th percentile of the depth of poverty distribution he was deemed poor.

Lovell et. Al. (1994) took a different approach to welfare measurement by employing distance functions and Deutsch and Silber (2005) employed this in multivariate poverty analysis. It will be the basis of our approach here.

2. The Distance Function, The Lower Convex Hull and Multivariate Poverty.

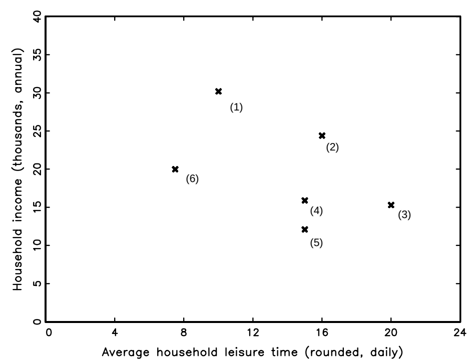
The distance function technique is borrowed from the production theory literature where it is used to measure efficiency. Consider a measure of the "distance" between a vector of the goods (functioning's and capabilities) of an agent and a comparison or yardstick vector, this approach seeks to measure the amount by which the household's set of attributes has to be scaled up or down so that it has the same well-being as the yardstick. This tool is called a *distance function* in the economics literature (Shephard (1953)) or a *gauge function* in the mathematics literature (Rockafellar (1970)). In mathematical notation this is:

$$D(\boldsymbol{x}_i, W) \equiv \min_d \{ d : W(d\boldsymbol{x}_i) = W^*, d > 0\},$$

where $\boldsymbol{x}_i$ is a vector listing a number of features of the i'th agent's circumstances, W is the chosen weighting function, W^* is the value of the weighting function for the yardstick and d is the distance measure which shows the minimum amount by which this observation's circumstances would have to be scaled up or down so that it would be on a par with the yardstick. The measure which comes out of this (d) will depend on $\boldsymbol{x}_i$, W and W^*.

If the objective is a measure of relative welfare then it makes sense to choose the yardstick to be the agent with either the lowest or highest well-being and to ask but how much do we need to scale back, or scale up, everybody else's set of attributes so that they have the same level of well-being as the yardstick? In order to make this operational a measure of well-being is required, essentially an aggregator function of the various agent characteristics that represents the agents welfare, an analogue of the classic utility function. Lovell et. al. (1994) and Deutsch and Silber (2005) use the translog function which is estimated by normalizing on one of the characteristics (it should be noted that if the true aggregator function is not homothetic, the rankings will not be independent of the normalizing characteristic chosen). Here and in Anderson et al (2005) we avoid this choice, to motivate what they propose consider the following example in which agents are represented by two indicators, leisure time (T) and income (Y) in a sample of six households. Figure 1 plots each household's circumstances

Figure 1



Now consider as an aggregate measure of well-being the geometric mean $T^{½}Y^{½}$. The worst off household (by *this* measure) is household (6) the best-off household is household (2). The curved lines on the left and right panels of Figure 2 show all of the combinations of our measured attributes which give exactly these levels of aggregate well-being.

Figure 2

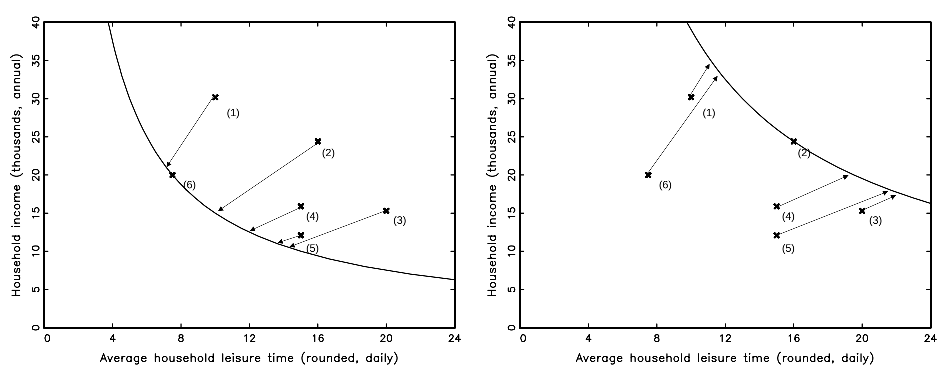


Figure 2 also shows the relative well-being indicators. The distance/gauge measures of relative well-being are given by the lengths of the arrow which connects each of the rest of the households to the reference welfare value curve. The welfare measures are listed in Table 1.

Table 1

Household	$D(\mathbf{x}_i, W)$ Low ref.	High ref
(1)	0.70	1.13
(2)	0.62	1.00
(3)	0.70	1.14
(4)	0.91	1.47
(5)	0.79	1.28
(6)	1.00	1.61

The distance measures in the left hand column are those from the left hand panel in Figure 2 (i.e. those where the worst off household is the reference household). Household (6) is the worst off: so their circumstances need only be multiplied by 1 (i.e. remain unchanged) for them to remain the worst off. Household (2) is the best off: their circumstances need to scaled back by the most (multiplied by 0.62) to reduce them to the same welfare value as (6). The figures in the right hand column are those which use the best off household as the reference (the worst off household (6) has to be scaled up by 61% in order to reach the reference level). Clearly since the two columns are based on the same welfare measure they agree on the ranking of the households. For reasons that will become clear later our attention will be focused upon the low reference point.

This approach is very easy to implement once you have chosen an aggregating function. Here $T^{½}Y^{½}$ was chosen but if $T^{0.75}Y^{0.25}$ had been chosen for example, household (1) would have been the household with the highest standard of living and the distances and ranking of the other households will be altered. Clearly the results depend upon data on individual's circumstances and the weighting formula. The problem lies in the dependence of the answers upon the weighting formula.

In standard models of consumer behavior the weighting function is essentially the agent's utility function rearranged in terms of income as a function of leisure for a given level of welfare. Typically the parameters of this function (the powers in our example) are not known and have to be recovered from estimated demand equations, largely because the level of individual welfare is not observed. This is indeed a problem when prices of the characteristics are not observed as well[2]. In addition it is hard to settle upon the specification of a demand system and corresponding utility function which satisfies the integrability conditions (i.e. homogeneity, symmetry, homotheticity and so on, see Deaton and Muellbauer (1980)) necessary for recovering the agents welfare function from its demand equations.

Anderson et. Al. (2005) avoids the need to choose a particular aggregation function or weighting scheme and removes the dependence of the final index on this crucial choice. Their approach considers *all* possible weighting formulae that have certain general properties and proposes a method that will calculate a *lower bound* on the distance measure of relative well-being which will be valid for all of them. The shared properties of W() entertained are:

Monotonicity: this means that the measured attributes are such that it is reasonable to expect that if the household had more of any of them, then their well-being would not decrease.

Quasi-concavity: this means that as the level of some measured attribute rises, well-being rises at a non-increasing rate which is closely related to inequality-aversion.

The measure is:

$$D(\mathbf{x}_i) \equiv \min_d \{ d : W(d\mathbf{x}_i) = W^*, d > 0, \text{for all monotone, quasi-concave } W\},$$

The basic intuition is that welfare level sets (i.e. sets like the curves in Figure 2) of any aggregator with these properties are convex to the origin; what is proposed is a simple way of calculating bounds on the set of all possible curves in a finite dataset which, following Rockafeller (1970), is approximated by the union of a set of closed half spaces as illustrated above.

Firstly let X denote a finite dataset, and let $conv(X)$ denote the lower convex hull of the data and let $mono(X)$ denote the upper monotone hull of the data. For our example data these objects are illustrated in the left and right panels (respectively) in Figure 3. Then, in the case where the reference household is the worst off in the dataset

$$D(\mathbf{x}_i) \equiv \min_d \{ d : d\mathbf{x}_i \in conv(X), d>0\}$$

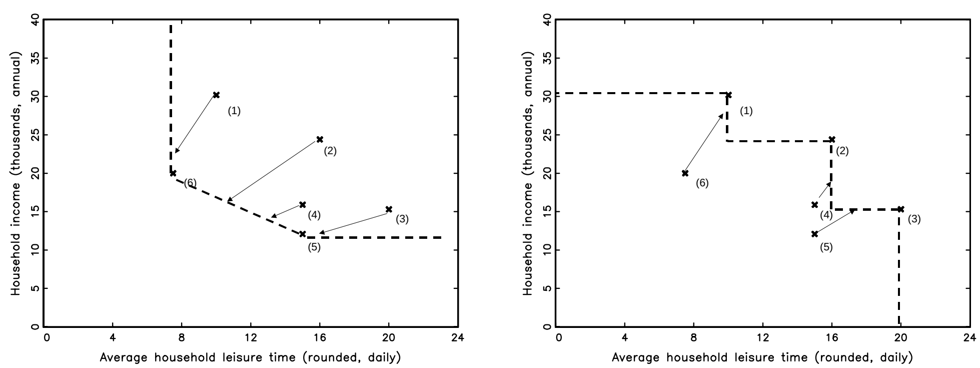
[2] This is less of a problem in production models because outputs and inputs are both directly observable.

And in the case where the reference is the best off

$$D(\boldsymbol{x}_i) \equiv \min_d \{ d : d\boldsymbol{x}_i \in mono(X), d>0\}$$

The resulting distance measures reflect the minimum amount one would have to scale each observation so that they shared equal ranking with the best and worst off observations. They represent lower bounds on these measures for *any and all* ways of choosing to weigh the various indicators you like as long as the weighting formula is monotone and quasi-concave. These two measures, for these data, can be seen in Figure 3. The left hand panel shows the lower convex hull of the data and the distances to it from each observation. Households (5) and (6) now tie for the ranking as worst off agent. None of the others can be the worse off (given monotoncity and concavity). In the right hand panel we show the upper monotone hull of the data. Now agents (1), (2) and (3) are all potential best off (for some increasing, concave weighting scheme) and so tie. The rest are not.

Figure 3



The resulting distance measures are given in Table 2. Together they show that agent (2) is the best off (it ties with (1) and (3)) in the right hand column as all are potentially the best off under some measure, but it is the agent which has to be deflated most when compared to the worst off. Similarly household (6) is the worst off, it ties with (5) as potentially the worst off (left hand column) but compared to the best off it has to be scaled up by more than (5).

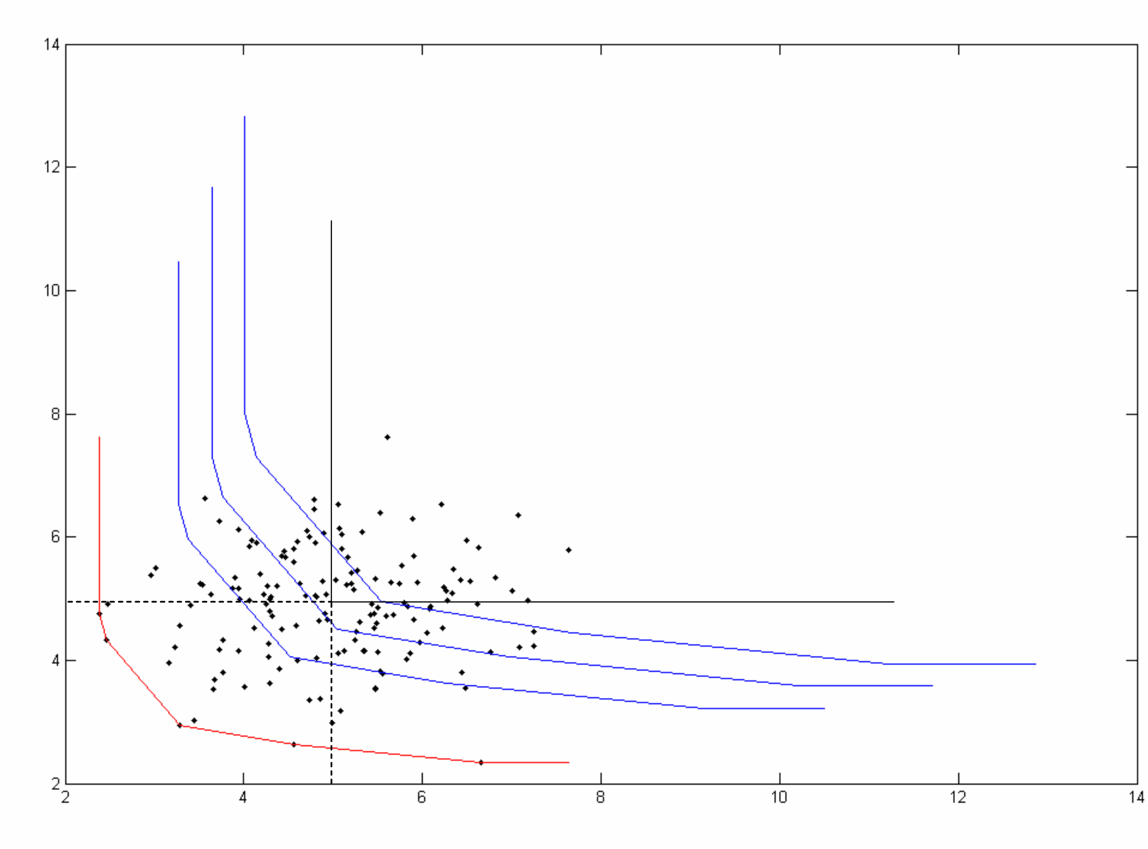
Table 2

Household	$D(\boldsymbol{x}_i)$	
	Lowest	Highest
(1)	0.75	1.00
(2)	0.68	1.00
(3)	0.79	1.00
(4)	0.88	1.07
(5)	1.00	1.22
(6)	1.00	1.23

The lower convex hull has a particularly useful interpretation in the case of analyzing poverty states, it is what we will term the Rawlsian Frontier, the set of potentially poorest agents in the sample, and consequently represents the frontier of poorest individuals in the population. Changes in the location of the frontier over time represent changes in the status of the poorest individuals. If for example the frontier in year one is everywhere below the frontier in a successive year then a Rawlsian welfare improvement may be deemed to have taken place.

We take as our poverty frontier a scaled up version of this Rawlsian Frontier, of course the scaling factor is to some extent arbitrary, but we could follow the relative poverty literature and define the frontier relative to the population. For example the median frontier would be defined by a scale factor that renders 50% of the population below that frontier. Here a choice of a boundary is defined by a specified poverty count, however once defined other insights into the nature of poverty and its progress can be gleaned from the magnitude of the radial distance of an agent from the boundary. Figure 4 presents a 2 dimensional example of 150 agents whose goods are distributed bivariate normal with means of 5, variances of 1 and covariances of 0. It shows the Rawlsian lower bound (lower convex hull) together with the 25%, 50% and 75% poverty frontiers (the respective scale factors were 1.3755, 1.5335 and 1.6850) together with individual good median cut-off points which define the union and intersection rules discussed in the introductory chapter. The rectangle defined by the dashed line defines the intersection set and the solid line right angle defines the upper bound of the union set.

Figure 4. The Lower Convex Hull, 25%, 50% and 75% poverty boundaries and median poverty cutoffs for two Goods which define the Union and Intersection sets.



In studying trends in poverty between two points in time (t_0, t_1) data from the two time periods can be pooled and a common Rawlsian frontier established together with common poverty boundaries. Then the specific year outcomes can be compared to these boundaries using any poverty index of choice.

3. Results.

Data from the World Bank on the life expectancy, literacy rate, school enrolment and gross domestic product per capita for 170 countries in the years 1997 and 2003 used in calculating the Human Development Index were collected. To get a flavour of the relative magnitudes of these variables summary statistics for the indicators for the year 2001 are reported in Table 3 (results for the other years were qualitatively very similar). It is of interest to note that the respective coefficients of variation, 0.1896, 0.2509, 0.2906 and 1.0561 are all relatively small except for the income variable. The marginal distributions are left skewed (mean < median) or dense in the upper tail for all but the income variable which is right skewed and hence dense in the lower tail.

Table 3. Summary Statistics for Human Development Index Data

Indicator	Mean	Median	Min	Max	St. Dev.
Life Expectancy (years)	65.4	69.8	33.4	81.3	12.4
Adult Literacy Rate	0.813	0.893	0.165	1.000	0.204
School Enrolment Rate	0.678	0.710	0.170	1.000	0.197
GDP per cap (PPP$US)	8564.8	5260	470	53780	9045.6

The common convex hull of the pooled sample was calculated and D1(t), t = 1997, 2003 computed which corresponds to deprivation relative to the pooled convex hull in year t . Clearly since the indices represent deprivation, the properties of their distributions can be examined to reflect world wellbeing given some welfare criterion.

Table 4 Deprivation Indices Summary statistics.

Variable	Mean	Median	Std. Dev.	Max value	Min value
D1(1997)	0.5837	0.5181	0.1464	1.0000	0.4435
D1(2003)	0.5830	0.5126	0.1529	1.0000	0.4316

Note that the deprivation distributions are right skewed (Mean > Median). Recall also that lower values of the location statistics in 2003 suggest welfare improvements. These location shifts came with an increase in the dispersion of the indices (both the range of the index and its standard deviation increased, implying greater inequality over the period.

Membership of the pooled convex hull corresponds to membership of the Rawlsian Frontier or "Poorest Countries Club", the membership was:

Bhutan (1997)
Ethiopia (1997)
Niger (1997)
Sierra Leone (1997)

Central African Republic (2003)
Niger (2003)
Sierra Leone (2003)
Zambia (2003)

Notice that the club membership is made up entirely of African nations. For an unequivocal Rawlsian welfare improvement the boundary should have been defined only by 1997 observations (since it would then be possible to assert that the lot of the poorest agents had improved). This clearly did not happen. For two club members (Bhutan and Ethiopia) things improved by 2003 in that they were no longer members of the "Rawlsian" club , for two club members things got worse in that they deteriorated to the boundary in 2003 and two club members (Niger and Sierra Leone) were part of the boundary in both years.

From an empirical perspective the fact that certain agents remained or joined the boundary may be purely a statistical artifact, a consequence of sampling error. This issue can be resolved by studying the properties of the deprivation distributions that underlay the data. Differences in the distributions of deprivation indices can be assessed by focus on the welfare function W(-D) which is assumed to be constant over time periods and, given f_{1997}(-D) and f_{2003}(-D) correspond to the respective distributions of -D, interest centers on the expected value of the change in welfare given by that:

$$E(\Delta W(-D)) = \int_{-\infty}^{x} W(-D)(f_{2003}^{i}(-D) - f_{1997}(-D))dD$$

Necessary and sufficient conditions for the change in welfare to be non negative depend on the nature of W( ), so that for W(x) with $(-1)^{j-1}d^{j}W/dx^{j} > 0$ j = 1,..,i for some i > 0 the i'th order stochastic dominance conditions are that:

$$\int_{-\infty}^{x} (F_{2003}^{i-1}(z) - F_{1997}^{i-1}(z))dz \le 0 \; for\ all\ x$$

with strict inequality holding for some x. and where, letting $f(x) = F^{0}(x)$, $F^{i}(x)$ is defined recursively as:

$$F^{i}(x) = \int_{-\infty}^{x} F^{i-1}(z)dz$$

Note that i'th order dominance implies j'th order dominance for any j > i. Notable welfare functions are i=1 Utilitarian Social Welfare with indifference to inequality, i = 2 expresses social preferences for more equality for a given level of average deprivation, i = 3 expresses social preferences for skewing the distribution away from extreme levels of deprivation at given levels of average deprivation and inequality and so on. Essentially higher orders of dominance attach greater weight to the deprived so that infinite order dominance attaches all weight in the social welfare function to the poorest individual.

Tests for these conditions are provided in Anderson (1996) and Davidson and Duclos (2000) which involve simultaneous comparisons of empirical counterparts of the functions defined above over a range of x's. Barrett and Donald (2003) provide tests based upon the maximum distance between the functions over the range of x. Here the Davidson and Duclos tests are employed, the simultaneous comparisons can be made using the studentized "t" distribution with tables available in Stoline and Ury(1979) or the Wald criteria developed in Wolak(1987) can be employed (see for example Anderson (2003)). Since a panel of 170 countries is being used the, observations years between the two years cannot be deemed to be independent (in fact the $\chi^2(16)$ test for independence of the two samples was 337.94 with an upper tail probability of 3.3690989e-062) so that allowance for the lack of independence has to be made (Davidson and Duclos (2000) and Anderson (2003) provide details of how this is done). To establish dominance the test must be performed in two parts, that is to say the comparison vector must have at least one significantly negative element and no significantly positive elements.

The range of D was partitioned into 5 equi-probable intervals based upon the pooled sample and the comparisons for i=1 and 2 are reported in Table 5 below. Establishing the quintile break points is equivalent to establishing the scale factors for poverty lines which yield 20%, 40%, 60% and 80% of the pooled sample in the poverty group respectively. The scale factor is the value by which the Rawlsian frontier (-D = -1 in this case) must be scaled to yield a boundary below which the corresponding proportion of the pooled sample resides so that 20% of the pooled sample has a relative deprivation greater than 0.7084, 40% of the pooled sample had a relative deprivation greater than 0.5282 and so on. The quintile proportions indicate the proportion of the sample below the respective poverty boundary in a given year so that 21.76% of the 2003 sample were below the 20% frontier and 18.24% of the 1997 sample were below the 20% frontier[3]. Notice that for all

[3] Closer examination of the data indicates that this reflects the demise of the African nations over the period, evidence elsewhere (Anderson (2005)) suggests that deterioration in the life expectancy index is a prime factor in this instance.

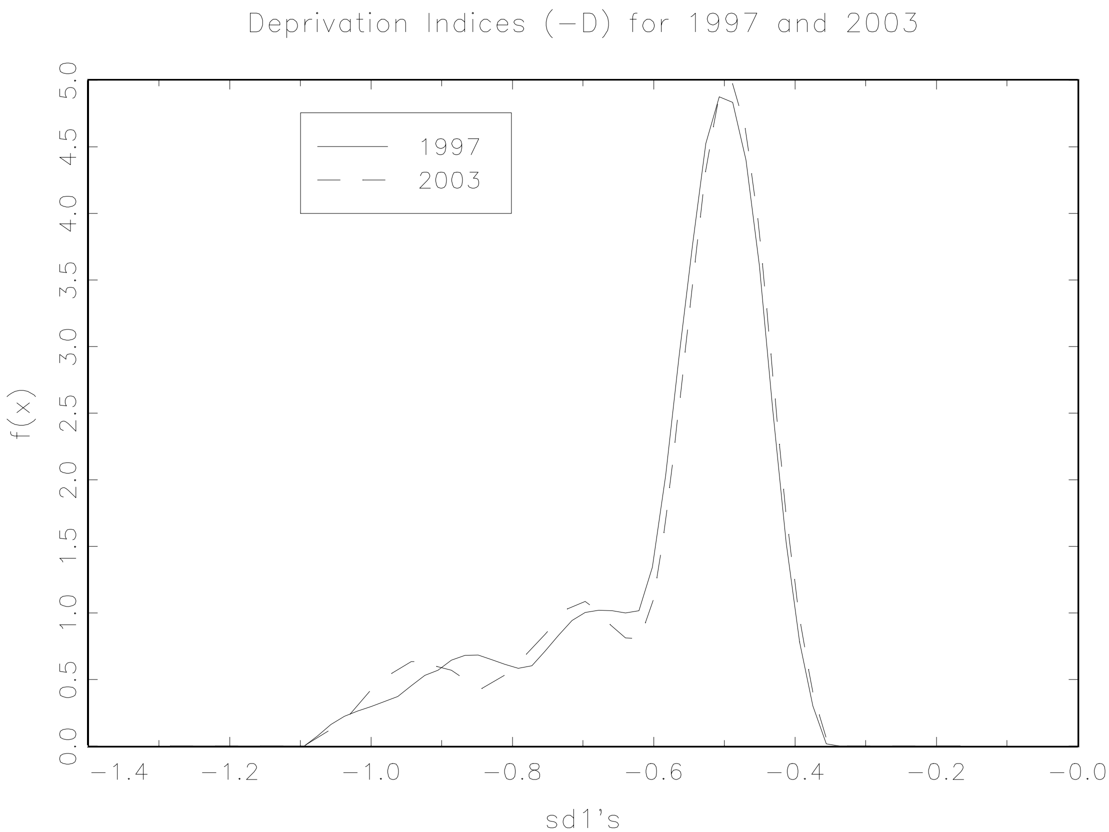
successive frontiers there were fewer countries below the frontier in 2003 than in 1997. In a similar fashion the Average deprivation reports the average value of the deprivations below each boundary which corresponds to a FGT(0) index. The “t” tests for differences report the test of H_0 that $FGT(i)_{1997}-FGT(i)_{2003} \leq 0$, i = 0,1 for the corresponding poverty boundary, as my be observed the hypothesis is never rejected at the 2.5% level (1.96) whereas $H_{0:}$ $FGT(i)_{1997}-FGT(i)_{2003} \geq 0$ is on the 80% boundary for FGT(0) and for the 100% boundary for FGT(1).

Table 5.

Pooled quintile and poverty line scale factors (for –D).		Quintile proportions (FGT(0) indices) 2003	1997	Average deprivation (FGT(1) indices) 2003	1997	“t” tests for differences FGT(0)	FGT(1).
0.2	0.7084	0.2176	0.1824	0.8453	0.8397	1.9178	1.9178
0.4	0.5282	0.3941	0.4059	0.7195	0.7314	-0.6332	1.3495
0.6	0.5063	0.5765	0.6235	0.6488	0.6630	-1.8059	-0.2645
0.8	0.4773	0.7706	0.8294	0.6096	0.6205	-3.2596	-1.6556
		1.0000	1.0000	0.5830	0.5837	0.0000	-2.2836

The Wolak (1987) Wald Criteria for the composite hypothesis $f_{1997}(x)$ first order dominates $f_{2003}(x)$ was 14.6414 with an upper tail probability of 0.0610 and the corresponding criteria for $f_{2003}(x)$ first order dominating $f_{1997}(x)$ was 3.6847 with an upper tail probability of 0.8107, which favours the first order dominance of 1997 by 2003. The corresponding criteria for the composite hypothesis $f_{1997}(x)$ second order dominating $f_{2003}(x)$ was 26.3756 with an upper tail probability of 0.0005 and the corresponding criteria for $f_{2003}(x)$ first order dominating $f_{1997}(x)$ was 8.5817 with an upper tail probability of 0.2246. Thus there is weak evidence for rejecting the notion of a welfare improvement and strong evidence for rejecting the notion of welfare deterioration. Further since dominance at order j implies dominance at higher orders and since a Rawlsian welfare improvement is equivalent to infinite order stochastic dominance, an unambiguous welfare improvement in a Rawlsian sense may be inferred.

A clearer idea of the relationship between the two deprivation indices may be gleaned from kernel density estimates for the corresponding years which are shown in diagram 1. The diagram has been cast in terms of –D so that it represents the distribution of a welfare index. .As may be seen there is some dominance indeterminacy at the lower extreme of the distribution which is reflected both in the dominance results and the corresponding FGT indices. This has been alluded to earlier as the consequence of the demise of the African nations which inhabit the lower tail of the distribution.



4. Conclusions.

A deprivation index relative to the lover convex hull of the joint distribution of a collection of characteristics or goods has been constructed which provides some insights into the notion of multivariate relative welfare and poverty. The index is essentially a lower bound of the potential set of welfare indices that obey monotonicity and quasi concavity axioms and avoids the specification of a weighting scheme for the various characteristics. The lower convex hull itself has a useful interpretation in the poverty context as the "Rawlsian" frontier, the set of agents corresponding to whom no poorer agents can be found. It enables simple univariate welfare and poverty comparisons to be made via stochastic dominance techniques provided one is not required to quantify the magnitude of a welfare or poverty change.

If quantification of the magnitudes of poverty is desired one is required to specify a poverty frontier if one wishes to avoid the strange consequences of union / intersection approaches to multivariate poverty. In the absence of an ability to estimate the parameters of multivariate welfare function (given a lack of prices or the willingness to make strong assumptions about the homotheticity of such a function with respect to one of the goods) a scaled up version of the Rawlsian frontier can be used in making comparisons between periods or between states. Then the vast range of poverty indices (see for example Zheng(1997)) can be employed on the indices.

These techniques were applied to World Bank data on the components of the Human Development Index for the years 1997 and 2003 for a panel of 170 countries. Membership of the Rawlsian Ppoverty Club was not confined to one particular year so that a Rawlsian welfare change could not be unambiguously inferred, however significant second order dominance results implied that evidence favoured a Rawlsian Improvement. FGT(0) and FGT(1) indices were computed and, excluding the extremely poor nations, significant improvements in the plight of poor countries was inferred.

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