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Multidimensional Poverty Measurement with Multiple Correspondence Analysis

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Louis-Marie Asselin
Université Laval and Institut de Mathématique Gauss – Canada

Multidimensional Poverty Measurement
with Multiple Correspondence Analysis

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[1] Associate researcher at CIRPÉE (Centre interuniversitaire sur le Risque, les Politiques Économiques et l'Emploi), Laval University, and director of IMG (Institut de Mathématique Gauss).

1. *Introduction*

This technical note aims to present succintly the use of a particular factorial technique, Multiple Correspondence Analysis (MCA), in the area of multidimensional poverty measurement. We will not come back on conceptual issues, on the choice of poverty indicators, and will not review other approaches found in the literature[2]. Neither will we present the statistical foundations of the basic factorial techniques, found in plenty of texbooks. We will rather highlight some characteristics of MCA which make it particularly attractive for the measurement of multidimensional poverty in view of poverty comparisons across space, time and socioeconomic groups.

We adopt a two-step approach, the first being the construction of a composite indicator from multiple primary poverty indicators, the second being the computation of poverty indices with the composite indicator. Obviously we focus here on the first step, the most challenging, the second being analogous to well-known techniques developed within the field of univariate moneymetric poverty analysis.

Numerical applications of our approach are given in some case studies presented in the third part of this same conference.

2. *Data reduction techniques, PCA and specificity of MCA*

Let's first introduce some notation.
On a population U of N population units U_i, K primary indicators I_k are measured, K>1. These indicators are possibly heterogenous in their nature:

- quantitative indicator, e.g. household income, number of bicycles, etc.
- qualitative or categorical
  - ordinal, e.g. level of education, etc
  - non ordinal, e.g. occupation, geographical region, etc.

We assume here that they are either quantitative or categorical ordinal.

The first step consists in **defining a unique numerical indicator C as a composite of the K primary indicators I_k, computable for each population unit U_i, and significant as generating a complete poverty ordering of the population U**.

Let's observe that the term "population unit" can refer as well to individuals and households as to villages, regions, countries.

For the discussion, it is important to clarify the terminology regarding the three concepts of **poverty indicator**, **poverty measure** and **poverty**

[2] See Asselin L.-M. and Dauphin A. (1999), Asselin L.-M. (2002)

index. Let I_{ik} be the value of indicator I_k for the elementary population unit i. I_{ik} is then properly a **poverty indicator** value. The value I_{ik} can be transformed as $g_k(I_{ik})$, with the function g_k, to better reflect a poverty concept relative to indicator I_k. This is frequently the case especially with a quantitative indicator I_k to which is associated a poverty threshold (poverty line) z_k. In that case, well-known transformations are $g_k(I_{ki})=(z_k- I_{ik})^a$. Then, $g_k(I_{ik})$ is called a **poverty measure** value, again defined on each elementary population unit. In the particular and also frequent case where the function g_k is the identity function, the poverty indicator and the poverty measure are the same. Finally, poverty measure values can be aggregated over the units for the whole population U, as $H_k\{g_k(I_{ik}), i=1, N\}$. Then H_k is called a **poverty index** relative to the indicator I_k for the population U. Obviously, this index H_k can be defined on sub-populations.

It is important to keep in mind that the three concepts indicator, measure and index are **relative** to the definition given of population unit, so that a household-based index, for example, can be considered as a village indicator when considering a population of villages for which this index exists, and so on.

A **composite poverty indicator** C takes the value $C_i(I_{ik}, k=1,K)$ for a given elementary population unit U_i.

2.1 Data reduction and PCA as a basic technique

Any composite indicator is necessarily a reductive variable since it tries to summarize K variables into just one. Statistical methods known as "factorial" techniques have precisely been designed since a long time as efficient data reduction techniques, essentially descriptive, whence the idea to look at them as potentially appropriate for solving the problem of our first step.

The basic optimal data reduction process comes from the Principal Component Analysis. Essentially, it consists in building a sequence of uncorrelated (orthogonal) and normalized linear combinations of input variables (K primary indicators), exhausting the whole variability of the set of input variables, named "total variance" and defined as the trace of their covariance matrix, thus the sum of the K variances. These uncorrelated linear combinations are latent variables called "components". The optimality in the process consists in that the 1st component has a maximal variance I_1^2, and all subsequent components a have decreasing variances I_a^2 whose sum is the total variance of the K indicators. This total variance is also named the total inertia of the distribution of the K indicators. The stepwise reduction process just described, computationally equivalent to eigenvalues and eigenvectors identification, corresponds geometrically to a change in the cartesian axis system (translation and rotation) of the k dimension euclidean space R^k. It is neutral regarding the orientation of the factorial axis. The variances l_a^2 are in fact the eigenvalues relative to the factorial axis determined by the eigenvectors.The whole process relies on analysing the structure of the covariance matrix of the K initial variables.

The 1st component F_1 is an interesting candidate as the composite poverty indicator C, but it must satisfy obvious consistency conditions on which we come back below. C has the following expression for the population unit i:

$$C_i = \sum_{k=1}^{K} W^{1,k} I_i^{*k} \ . \qquad (1)$$

The I^{*k} are the standardized primary indicators. The factor score coefficients $W^{1,k}$ must have signs consistent with the interpretation of the 1st component as a poverty indicator. At the end of the process, it comes out that the $W^{a,k}$ are in fact the usual multiple regression coefficients between the component Fa and the standardized primary indicators. Built this way, the 1st component, if acceptable as the composite indicator, C, can be described as the best regressed latent variable on the K primary poverty indicators. No other explained variable is more informative in terms of poverty. The set of coefficients $W^{1,k}$, i.e. the weights given to the primary indicators, can be seen as expressing the social choice for poverty reduction, in terms of the (basic) goods measured by the K primary indicators.

Interesting as it is, the PCA technique has some limitations:

a) the whole technique has been developed for a set of quantitative variables, measured in the same units[3]. The optimal sampling properties for parameter estimation depends on the multivariate normal distribution and no more exist with qualitative variables;
b) the operationalization of the composite indicator, for population units not involved in the sample used for estimation, is not very appealing since weights are applicable to standardized primary indicators.

Since concepts of multidimensional poverty are frequently measured with qualitative ordinal indicators, for which PCA is not a priori an optimal approach, looking for a similar but more appropriate factorial technique is justified. Here comes naturally into the picture Multiple Correspondence Analysis (MCA), designed in the sixties-seventies precisely to improve the PCA approach, when this one looses its parametric estimation optimal properties.

[3] T.W. Anderson (1958), p. 279.

2.2 Specificity of MCA

From now on, we will assume that the K primary indicators are categorical ordinal, the indicator I_k having J_k categories. It's a very general setting, applicable to any mix of quantitative and qualitative poverty indicators, since a quantitative variable can always be redefined in terms of a finite number of categories. Let's associate to each primary indicator I_k the set of J_k binary variable 0/1, corresponding each to a category of the indicator. We introduce the following notation:

- X(N,J) : the matrix of N observations on the K indicators decomposed into J_k variables, where $J = \sum_{k=1}^{K} J_k$ is the total number of categories. X is named the indicatrix matrix.
- N_j : the absolute frequency of category j, i.e. the sum of column j of X
- N': the sum of the elements of matrix X, i.e. NµK
- $f_j = \frac{N_j}{N'}$: the relative frequency of category j
- $f_j^i = \frac{X(i,j)}{X(i)}$, where X(i) is the sum of line i of the matrix X. The set $f_J^i = \left\{ f_j^i, j = 1, J \right\}$ is named the profile of observation i.

MCA is a PCA process applied to the indicatrix matrix X, i.e. to the set of the J binary variables in the R^N space, but with the χ^2 - metric on row/column profiles, instead of the usual Euclidean metric.

The χ^2 - metric is in fact a special case of the Mahalanobis metric developed in the thirties and used in Generalized Canonical Analysis. It takes here the following form, for the distance between two observation profiles i and i' in the R^J space:

$$d^2\left(f_J^i, f_J^{i'}\right) = \sum_{j=1}^{J} \left(\frac{1}{f_j}\right)\left(f_j^i - f_j^{i'}\right)^2 . \qquad (2)$$

The only difference with the Euclidean metric lies in the term $\left(\frac{1}{f_j}\right)$, by which smaller categories receive a higher weight in the computation of distance.

The difference between MCA and PCA shows up particularly in two properties which seem highly relevant for the poverty meaning of the numerical results.

Property #1 (marginalization bias)

MCA is overweighting the smaller categories within each primary indicator. In fact, we have:

$$W_{j_k}^{\alpha,k} = \frac{N}{N_{j_k}^k} \text{Covariance}\left(F_\alpha, I_{j_k}^k\right) \quad (3)$$

where

$W_{j_k}^{\alpha,k}$ = the score of category j_k on the factorial axis α (non – normalised)

$I_{j_k}^k$ = the binary variable 0/1 taking the value 1 when the population unit has the category j_k.

F_α = the score (non - normalized) on the factorial axis α

$N_{j_k}^k$ = the frequency of the category j_k of indicator k

Thus, in the case of a binomial indicator, the marginal category will receive a higher weight, since the covariance is the same for both categories.

In terms of poverty, if we think of (extreme) poverty in a given society as being more relative than absolute and characterized by social marginalization, i.e. by the belonging to a minority group within the population, the group of people characterized by a poverty category j_k, then this category will receive more weight or consideration in the computation of a composite indicator of poverty. If, as above, we interpret the factorial weights (regression weights) as expressing the social choice in poverty reduction, then these highly weighted poverty attributes represent those which this society try to eliminate in priority.

Property #2 (reciprocal bi-addivity) or (duality)

The way it is defined, MCA can be applied on the indicatrix-matrix either to the row-profiles (observations) as to the column-profiles (categories), and then it has the following remarkable and unique duality property:

$$F_\alpha^i = \frac{\sum_{k=1}^{K} \sum_{j_k=1}^{J_k} \frac{W_{j_k}^{\alpha,k}}{\lambda_\alpha} I_{i,j_k}^k}{K} \quad (4a)$$

where

K = number of categorical indicators

J_k = number of categories for indicator k

$W_{j_k}^{\alpha,k}$ = the score of category j_k on the factorial axis α(non – normalised)

I_{i,j_k}^{k} = the binary variable 0/1 taking the value 1 when the unit i has the category j_k.

F_{α}^{i} = the score (non - normalized) of observation i on the factorial axis α

and reciprocally

$$W_{j_k}^{\alpha,k} = \frac{\sum_{i=1}^{N_{jk}} \frac{F_{\alpha}^{i}}{\lambda_{\alpha}}}{N_{j_k}^{k}} \qquad (4b)$$

Let's assume, for example, that the first factorial axis meets the consistency conditions to be considered as a poverty axis[4] and that we can take as the composite indicator of poverty $C_i = F_1^i$.

Then the duality relationships stipulate:

(4a): the composite poverty score of a population unit is the simple average of the factorial weights (standardized) of the K poverty categories to which it belongs.

(4b): the weight of a given poverty category is the simple average of the composite poverty scores (standardized) of the population units belonging to the corresponding poverty group.

We think that these both properties, and especially (4b) for the reciprocal bi-addivity, are quite relevant for the poverty meaning of the numerical results coming out of this specific factorial analysis, MCA. That's why we explore more attentively in the following section a research strategy in applying MCA to the problem of measuring multidimensional poverty.

3. MCA applied to defining a composite indicator of multidimensional poverty

3.1 A fundamental consistency requirement

We have now to look more closely to the conditions under which the factorial approach, and especially MCA, can really generate a relevant composite indicator of multidimensional poverty. We could have here a full axiomatic formulation so that the objective of poverty comparison is satisfactorily met. But with a two-step approach, the axiomatic requirements can be largely simplified. If the first has provided a relevant

[4] We come back below on these consistency conditions.

composite poverty indicator, the axiomatic requirements for the second step, regarding the computation of aggregated poverty indices, can rely on standard requirements now generally accepted in the case of unidimensional poverty measurement, especially for the well-known case of moneymetric poverty. For the first step, the construction of a composite indicator C from K ordinal categorical indicators I_k, there is at least the following requirement:

<u>Monotonicity axiom</u> (M)[5]

> The composite poverty indicator must be monotonically increasing in each of the primary indicators I_k.

The axiom just means that if a population unit i improves its situation for a given primary indicator I_k, then its composite poverty value C_i increases: its poverty level decreases.

Let's see what it means if we intend to take the first factorial component F_1 as the composite poverty indicator C. From (4a) above, its expression would then be:

$$C_i = F_1^i = \frac{\sum_{k=1}^{K}\sum_{j_k=1}^{J_k}\frac{W_{j_k}^{1,k}}{\lambda_1}I_{i,j_k}^k}{K} \qquad (5a)$$

To simplify, let's write $W^{*\alpha,k} = \dfrac{W^{\alpha,k}}{\lambda_\alpha}$ for the normalized category-score on the factorial axis a. Then we have:

$$C_i = \frac{\sum_{k=1}^{K}\sum_{j_k=1}^{J_k}W_{j_k}^{*1,k}I_{i,j_k}^k}{K} \quad , \qquad (5b)$$

The monotonicity axiom translates into two requirements:

<u>M1: First Axis Ordering Consistency (FAOC-I)</u> for indicator I_k

> For any indicator I_k, for which the ordering relation between categories is noted $<_k$, the ordering relation $<_w$ of the weights $W_{j_k}^{*1,k}$ must be equivalent to either $<_k$ or to $>_k$.

M2: <u>Global First Axis Ordering Consistency (FAOC-G)</u>

[5] We assume that the sign of the composite indicator is chosen (see above the neutrality of the factorial process for axis orientation) so that a larger value means less poverty, or, equivalently, a welfare improvement and that the ordering relation A < B between two categories A and B of the same indicator means that B is preferable to A.

For all indicators I_k, the FAOC-I condition is fulfilled with the same orientation: the ordering relation $<_w$ is equivalent to either $<_k$ for all indicators or to $>_k$ for all.

If and only if the monotonicity axiom is satisfied can $C = F_1$ be taken as a composite poverty indicator. But then the reciprocal bi-addivity property of MCA gives a very interesting consistency result for C_i. Due to (4b) which says that the weight of an indicator category, $W_{j_k}^{1,k}$, is given by the average composite poverty score of the population group of size N_{j_k} having the category (attribute) j_k, we can state the following property of C :

Composite Poverty Ordering Consistency (CPOC)

With $C = F_1$ acceptable as composite poverty indicator, let the population group P_{j_1} have a category j_1 of I_k inferior to category j_2 possessed by the group P_{j2}. Then the group P_{j_1} is also poorer than P_{j2} relatively to the composite poverty.

In other words, the population ordering for a primary indicator I_k is preserved with the composite indicator. This is a remarkable consistency property specific to MCA, due to the dual structure of the analysis.

Clearly, there is no guarantee that MCA runned on the K primary indicators will come out with the FAOC property, and then using the first factorial component as the composite poverty indicator would be inconsistent and not acceptable. In fact, everything depends on the structure of the covariance matrix X'X[6].

There are two ways of overcoming this unpredictable difficulty: minor adjustments to the set of the K primary indicators, or exploiting more than one factorial axis.

3.2 Adjustments to the set of the K primary indicators

It should be noticed first that a binomial indicator meets always the FAOC-I requirement. For a multinomial indicator not satisfying this requirement, sometimes regrouping some categories can achieve the FAOC-I. If this operation does not succeed, a more radical one is to eliminate the indicator. Obviously, if the primary indicators have been carefully selected, defined and tested, this is a high price to pay just for satisfying a technical condition. We do not favour the elimination of indicators, but it becomes more acceptable

[6] We write here X for the matrix of centered variables.

when the number of indicators K is large and there appears some duplication in a specific area of poverty.

If all indicators satisfy FAOC-I, but FAOC-G is not met, it means that relatively to the first factorial axis there are two subsets of indicators with opposite ordering on this axis, thus negatively correlated. Two such disjoint subset of indicators will always be the situation with K binomial indicators, the situation met particularly with the now very popular approach of asset poverty measurement. In this last case, there is no consistency problem if one of the two subsets is the empty subset «, which is not unusual. Let's assume that both subsets are not empty. It means that the multivariate measurement of poverty cannot be shrunk into an unidimensional poverty measurement, and that in spite of existing correlations, the poverty concept reflected in the K chosen indicators is really deeply multidimensional. The only way to get out of this inconsistency would be to eliminate one of the two subset of indicators, which a priori does not seem acceptable: the information loss would then be too important. We need a more appropriate research strategy.

3.3 A research strategy using more than the first factorial axis

We need some more tools to design a research process more developed than considering just the first factorial axis. Let L be the number of factorial axis, determined by the rank of the matrix X. We have L§ J-K, where J is the total number of categories for the K indicators.

Let $\Delta_l^k = \dfrac{\sum_{j_k=1}^{J_k} N_{j_k}^k W_{k,j_k,l}^2}{N}$ (6)

be the <u>discrimination measure</u> of indicator I_k on the factorial axis *l*. It is in fact the variance of the distribution of the categorical weights on axis *l*, since the average weight is always 0.

We know from the theory of MCA that

$$\lambda_l^2 = \frac{\sum_{k=1}^{K} \Delta_l^k}{K} \qquad (7),$$

i.e. the eigenvalue of axis *l* is the average of the discrimination measures.

It follows from the basic factorial equation

$$\text{Total Inertia} = I_{tot} = \sum_{l=1}^{L} \lambda_l^2 \qquad (8)$$

that we have the equation below:

Total Inertia Decomposition

$$I_{tot} = \frac{\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{j_k=1}^{J_k} N_{j_k}^{k} W_{k,j_k,l}^{2}}{K \times N} = \frac{\sum_{l=1}^{L}\sum_{k=1}^{K}\Delta_l^k}{K} \qquad (9)$$

In the case of MCA, it is shown that $I_{tot} = \frac{J}{K} - 1$, i.e. the average number of categories per indicator, minus 1. If all indicators are binomial, the Total Inertia is precisely 1.

Let's also write $\kappa = \{1,2,....,K\}$, the set of integers from 1 to K.

We will now generalize the preceding approach to the composite poverty indicator.
For each factorial axis *l*, we can idenfify one or more subsets of indicators, each subset satisfying the Axis Ordering Consistency condition (AOC), i.e. both requirements AOC-I (4a) and AOC-G (4b), which now no more refer only to the first axis. The worst situation is when, for a given axis *l,* no indicator meets AOC-I, and then there is just one subset, the empty subset «. Among these AOC subsets, we retain the one whose sum of discrimination measures is maximal. We will then consider that there is a poverty dimension specific to axis *l* if and only if the sum of discrimination measures of this AOC-subset represents the larger part of the total discriminating power of axis *l*, i.e. is larger than 50% of $K \times \lambda_l^2$. To each factorial axis *l*, we can thus associate a unique subset of the K indicators, whose indices are a subset κ_l of κ, so defined:

Poverty Dimension Set of axis *l*

The Poverty Dimension Set of the factorial axis *l*, $\{I_k\}_{k \in \kappa_l}$, is the most discriminating subset of AOC indicators satisfying $2 \times \sum_{k \in \kappa_l} \Delta_l^k > K\lambda_l^2$.

It should be clear that the set $\{I_k\}_{k \in \kappa_l}$ can be empty, which means that the factorial axis *l* does not represent any poverty dimension.

It should be clear also that the poverty dimension sets are not necessarily disjoint: an indicator can belong to many of them. The potential intersection between these sets can be eliminated by a sequential process starting with the first axis and continuing with the others as ordered by MCA, since the discriminating power of each axis is decreasing. The way to eliminate these intersections, while trying to retain at each step the maximal inertia, is naturally coming out of the total inertia decomposition (9): at each step, we keep a given indicator k into the poverty dimension set where its discrimination measure is larger. We refer to this sequential process as to the algorithmic identification of independent poverty dimensions, more simply the poverty dimensions algorithm. Let then $\kappa_l^* \subseteq \kappa_l$ be the subset of indicator indices at step L*¥1 in the sequential process.

Normally, to insure that the process retains a maximal proportion of I_{tot} in the disjoint poverty sets, the algorithm must be pursued until L*=L. We then have built a complete sequence of poverty dimension sets.

Complete sequence of poverty dimension sets

The sequence of disjoint subsets of indicators $\{I_k\}_{k \in \kappa^*_l}$ resulting from the application of the poverty dimensions algorithm until L*=L, is called a complete sequence of poverty dimensions sets. The number *d* of non empty subsets is the number of independent poverty dimensions provided by the set of the K primary indicators.

Two cases are then possible: all K indicators belong to the sequence, i.e. $\bigcup_{l=1}^{L} \kappa_l^* = \kappa$, or some indicators are not retained from the process. In this last case, they should simply be eliminated from the search of a composite indicator: on no factorial axis they meet the minimal consistency requirement.

The poverty dimensions algorithm can rapidly become quite demanding with a large number K of primary indicators, let's say K¥ 10, which is not unusual in applied multidimensional poverty. As an example, with 10 indicators having in average 3 categories, the process could involve the analysis of L=20 factorial axis. Even if all well-known softwares allow such an analysis, with some tedious work for the analyst, to facilitate the operationalization, it seems better to introduce the possibility of interrupting the algorithm when some kind of ideal situation is met, we mean when all K indicators appear in a sequence of disjoint poverty sets. We are thus taken to the following definition:

Minimal sequence of complete poverty dimension sets

A minimal sequence of complete poverty dimension sets is obtained when the poverty dimensions algorithm is interrupted at the smallest value L*§L for which either $\bigcup_{l=1}^{L^*} \kappa_l^* = \kappa$, i.e. all indicators are included in the sequence of disjoint poverty sets, or L*=L.

Here also, the number *d* of non empty subsets is the number of independent poverty dimensions provided by the set of the K primary indicators.

It should be noticed that this definition allows, in particular, for stopping the process to the 1st factorial axis if is met the situation considered above, i.e. when the FAOC condition is achieved.

We can now give, from (5a), a generalized definition of the composite poverty indicator, which can be applied when the first factorial axis does not meet the FAOC requirement.

Generalized definition of the composite poverty indicator

Let a minimal sequence of complete poverty dimension sets be obtained, which is always possible with the poverty dimensions algorithm. Then the value C_i of the composite poverty indicator for the population unit i is given by:

$$C_i = \frac{\sum_{l=1}^{L^*} \sum_{k \in \kappa_l^*} \sum_{j_k=1}^{J_k} \frac{W_{j_k}^{l,k}}{\lambda_l} I_{i,j_k}^{k}}{K} \qquad (10)$$

Definition (5a) is the special case where L*=1: all K indicators belong to the poverty dimension subset of the factorial axis 1. This is the case where the multivariate measurement of poverty can be logically reduced to one aggregate poverty dimension, due to the structure of the correlation matrix: all K indicators are positively correlated. In the general case, there is more than one poverty dimension, in fact one for each poverty set, and the way to aggregate them is suggested by the structure of (5a) and the fundamental equation of decomposition of the total inertia (9): instead of picking up the J_k weights attributed to the indicator I_k only from the set of weights provided by the first factorial axis, it takes them from the axis which define the poverty dimension subset to which it belongs.

It should be remarked that the two very relevant properties of MCA, the marginalization bias (3) and the reciprocal bi-additivity, especially (4b), are valid in each of the L* axis involved in the generalized definition and thus keep their meaning, in the relevant poverty dimension l, for the interpretation of the categorical weights of the κ_l^* indicators defining this dimension. Also, the composite poverty ordering consistency remains valid for each identified poverty axis, with obvious adaptation.

The whole generalization approach must be viewed as an effort to not obliterate, rather to highlight, the deep multidimensional poverty structure hidden in the K-variate measurement of poverty, and at the same time to integrate into the composite poverty indicator a maximal information from the full information contained in the K primary indicators, as measured by the Total Inertia.

4. *Conclusion*

We have tried to highlight here some relevant characteristics of a specific factorial technique, Multiple Correspondence Analysis (MCA), in view of constructing a composite poverty indicator from K primary indicators. This is the first of a two-step approach to indices of multidimensional poverty. The context deliberately chosen refers to an unspecified number K¥2 of categorical-ordinal poverty indicators, a quite frequent situation in the area of multidimensional poverty measurement.

Relatively to our problem, the common feature of factorial techniques consists in taking the first factorial component as the composite poverty indicator. It is the best latent variable regressed on the primary indicators, and thus, under the restriction of consistency, the corresponding factor-score coefficients (indicator weights) can be seen as expressing the social choices of a given population in trying to get out of poverty. Starting from the basic characteristics of factorial analysis well-known from the founding technique, Principal Component Analysis (PCA), but also from the fact that PCA has been developed essentially for quantitative variables, the main MCA characteristics here highlighted are:

a) Property #1: the marginalisation bias, expressed in equation (3) for the categorical weights,
b) Property #2: the reciprocal bi-additivity, or duality, expressed in equations (4a) and (4b).
   From these two properties, categorical MCA weights receive an interesting and relevant meaning in terms of poverty groups.
c) The Composite Poverty Ordering Consistency coming out of the FAOC requirement (First Axis Ordering Consistency) resulting from the Monotonicity Axiom.

In view of preserving as much information as possible when the FAOC requirement is not met, we propose here an algorithmic process, the algorithmic identification of independent poverty dimensions, which allows a generalization of the preliminary definition of the composite indicator based uniquely on the 1st factorial axis. This process makes explicit the multidimensional poverty structure of the K primary indicators, by the identification of *d* disjoint non-empty subsets of poverty indicators, defining as many independent poverty dimensions. The algorithm operationalizes in a systematic and finite computation process the well-known power of all factorial techniques in terms of graphical analysis as revealing the deep structure of multidimensionality.

The main danger which threatens factorial approaches to multidimensional poverty measurement is a simplistic treatment intentionally reduced to the first factorial axis only, since it could imply a high information loss. We think that the generalised MCA definition proposed in equation (10) helps to overcome this threat and constitutes an honest and relevant candidate as a composite indicator of multidimensional poverty.

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International Poverty Centre
SBS – Ed. BNDES, 10º andar
70076-900 Brasilia DF
Brazil

povertycentre@undp-povertycentre.org
www.undp.org/povertycentre
Telephone +55 61 2105 5000