Multidimensional Poverty: A Comparison between Egypt and Tunisia

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7th July 2005

A preliminary version

Abstract

It is common to argue that poverty is a multidimensional issue. Yet few studies have included the various dimensions of deprivation to yield a broader and fuller picture of poverty. The present paper considers the multidimensional aspects of deprivation by specifying a poverty line for each aspect and combines their associated one-dimensional poverty-gaps into multidimensional poverty measures. An application of these measures to compare poverty between Egypt and Tunisia is illustrated using robustness analysis and household data from each country.

Keywords: Multidimensional Poverty Indices; Robustness Analysis; Egypt; Tunisia.

JEL Classification: D31; D63; I32.

This paper is prepared for the international conference on multidimensional poverty organized by the International Poverty Center, United Nations Development Programme, 29-31 August, 2005, Brasilia. I am grateful to Jean-Yves Duclos and Stephen Younger for their useful comments.

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1 Introduction

There is a widespread agreement that poverty is a multidimensional issue, including a number of monetary and non-monetary deprivations. For instance, the basic needs approach of Streeten (1981) perceives development as an improvement in an array of human needs and not just as growth of income. To some extent, it is sometimes true that income rise enables households to better reach their basic needs. This fact presupposes, however, the presence of markets for all basic needs which do not always exist. Further, empirical studies often reveal weak correlation between income and other welfare variables.[1]

Given the rather loose relation between income (or expenditure) and welfare in many contexts (like incomplete markets, presence of externalities and public goods), it is irrelevant to look solely at income distribution to assess the extent of poverty.[2] In recent pioneer papers, Sen (1985, 1992, 1999) suggests to measure welfare and poverty *directly* by observing individuals' *functionings* and *capabilities*, where *functionings* deal with what a person can do and *capabilities* indicate the freedom that a person enjoys in terms of *functionings*. Poverty indices have then to capture the inability of individuals to achieve a minimal level of capabilities to function (such as the inability to be healthy, well-nourished, educated, sheltered, etc.).[3]

The fact that it is hard in practice to obtain individual data on the main welfare variables has largely led researchers to follow an *indirect* approach, usually the monetary one, to measure poverty. However, since the beginning of the 1990s, data on attributes other than income and/or expenditures have become increasingly available. The multidimensional approach is thus more than ever required to better understand the performance of a given country in the combat against poverty in all its aspects.

Once the dearth of data availability has been overcome, researchers are confronted with a new challenge: How should the different attributes be integrated to yield a broader and fuller picture of poverty? Should this measure focus on the situation of those who are poor according to all attributes simultaneously, or

[1] On this, see for instance Klassen (2000) and Duclos et al. (2002).

[2] In reality, it is common in developing countries to base the monetary welfare measures on total expenditures rather than on income. See for instance Slesnick (1998). Thus, for short, we will often use the term ”income” instead of ”total expenditures”.

[3] The approach followed in this paper falls short of the *capability* approach suggested by Sen. In reality, the multidimensional poverty yardsticks capture here the *functionings* achieved and not the freedom to achieve them.

should it also account for the deprivation of those who do not reach the required minimum for any one attribute?

In considering the aggregation problem, a distinction is made between two different methods. The first sums across individuals, to form a synthetic index for all individuals in one dimension, and then combines all the one-dimensional indices to yield a multidimensional poverty measure.[4] The second combines the multiple indicators of deprivation at the individual level first, and then aggregates them across individuals into an overall social index.

Holding the universal conviction that multidimensional poverty measures are ethically and theoretically appealing, our purpose in this paper is to present the main contributions to this literature. A distinction is made between whether or not poverty measures are based on an axiomatic approach. An application of these measures to compare multidimensional poverty between Egypt and Tunisia is illustrated using household data.

Like in a one-dimensional setting, multidimensional poverty comparisons also require the specification of multidimensional poverty lines and measures, a procedure which is ethically and empirically highly controversial. Although the literature dealing with issues of dominance in multidimensional context is still in its embryonic stages, some dominance conditions under which distributions can be ranked have been derived by Bourguignon and Chakravarty (2002) and Duclos et al. (2002). Thus, it is possible to check whether multidimensional poverty comparisons between Egypt and Tunisia are robust to the multiple choices of poverty lines and poverty measures.

The rest of the paper is structured as follows. Section 2 develops the methodology followed by the UNDP (1997) to elaborate the human poverty index. Section 3 presents the theoretical framework of certain multidimensional poverty measures based on an axiomatic approach. Section 4 implements the developed methodologies to Egypt and Tunisia. Section 5 compares multidimensional poverty between Egypt and Tunisia using robustness analysis. Finally, Section 6 concludes.

[4] An example of this is the *Human poverty Index* of the UNDP (1997).

2 A Non-Axiomatic Approach: The Human Poverty Index

A simple way to study multidimensional poverty is to examine several welfare indices, separately. This path was followed by Adams and Page (2001), for example. Using aggregate data from the World Bank that is available for several countries in the Middle East and North Africa, these authors compare the performances recorded for each indicator in several countries in this region. They observe that there is no clear relationship between a reduction in monetary poverty and an improvement in other welfare indicators. A country may, for example, have a high rate of monetary poverty alongside a high rate of education, and vice versa. Multidimensional poverty comparisons are thus hard to complete unless all indicators are previously aggregated into an overall index.

The Human Development Report published by the UNDP (1997) states that, while pointing to a crucial element of poverty, a lack of income only provides part of the picture in terms of the many factors that impact on individuals' level of welfare (longevity, good health, good nutrition, education, being well integrated into society, etc.). Thus, a new poverty measure is called for—one that accounts for other welfare indicators, particularly:

1. An indicator that accounts for a short lifespan. Denoted HPI_1, this reflects the percentage of individuals whose life expectancy is less than 40 years.

2. A measure which is related to the problem of access to education and communications. The proportion of the adult population that is illiterate, denoted HPI_2, could be considered as an appropriate indicator.

3. A composite index capturing facets of the level of material welfare, HPI_3. This is computed as the arithmetic mean of three indicators: the percentage of the population without access to health care (denoted $HPI_{3,1}$), to safe water ($HPI_{3,2}$), and the percentage of children under age five suffering from malnutrition ($HPI_{3,3}$).

The proposed composite poverty index was elaborated by Arnand and Sen (1997). It is written as follows:

$$HPI = (w_1 HPI_1^\theta + w_2 HPI_2^\theta + w_3 HPI_3^\theta)^{\frac{1}{\theta}}, \tag{1}$$

with $w_1 + w_2 + w_3 = 1$ and $\theta \geq 1$.

When $\theta = 1$, the three elements of HPI are perfect substitutes. However, when θ tends to infinity, this index approaches the maximum value of its three components, i.e. $\max(HPI_1, HPI_2, HPI_3)$. In this event, the HPI will only fall if its highest-valued component decreases. These two extreme cases are difficult to advocate, so an intermediate value is sought for ordinal comparisons of poverty.

Estimates of the HPI, for $\theta = 3$ and $w_i = \frac{1}{3}$, have been performed by the UNDP (1997) for the developing countries which have the required data. These estimations show that human poverty affects more Egypt than Tunisia; with $HPI = 34.8$ for the former and 24.4 for the latter. These findings set this two countries at the middle of the ranking within the MENA regions. At the top of the ranking we find Jordan, United Arab Emirates and Libya. At the bottom are Sudan, Mauritania and Yemen.

Low levels of life expectancy, education, and health are of concern in their own right, but they merit special attention when they accompany monetary deprivation. The HPI omits, however, this dimension of poverty, which is at least as important as the aspects this index captures. Furthermore, this index does not account for the correlation that may exist between its components. For instance, an illiterate individual whose life expectancy is less than 40 years will be doubly counted. Another drawback of this widely used index is that it does not provide information on how attributes are distributed among the population. Therefore, it is possible to have improvements in the HPI while large segments of society stagnate or even worsen their situation. Finally, ordinal comparisons of poverty will be very sensitive to the (arbitrary) values assigned to w_i and θ. An alternative approach that allows for a better characterization of the weights assigned to each attribute would certainly be more appropriate.

3 An Axiomatic Approach to Measuring Multidimensional Poverty

Let $x_i, i = 1, 2, ..., n,$ be a vector of k basic needs of the ith person, X be a $(n \times k)$-matrix (whose ith row is x_i) summarizing the distribution of k attributes among n persons, and $z = (z_1, ..., z_k)$ be a k-vector of the minimum levels of basic needs. The most general form of a class of multidimensional poverty measures can be given by the following equation:

$$P(X, z) = F[\pi(x_i, z)], \tag{2}$$

where $\pi(\cdot)$ is an individual poverty function that indicates how the many aspects of poverty must be aggregated at the individual level. The function $F(\cdot)$ reflects the way in which individual poverty measures are aggregated to yield a global poverty index.

Measuring poverty always raises ethical questions. For example, should we consider a person who is well endowed with some attributes poor if she is unable to reach the minimum requirements for one basic need? The literature dealing with multidimensional poverty distinguishes between measures based on the *union* of the various aspects of deprivation from those based on their *intersection*.[5] If we measure poverty in the dimensions of expenditure and education, say, then the poor people would be those who have *either* low expenditure *or* low educational level. This is a *union* definition of multidimensional poverty and $\pi(\cdot)$ will be:

$$\pi(x_i, z) \begin{cases} = 0, & \text{if } x_{i,j} \geq z_j, \quad \forall j = 1, 2, \ldots, k, \\ > 0, & \text{otherwise,} \end{cases} .$$

Yet an *intersection* definition would consider as poor those who have low expenditure *and* low educational level. In such case, $\pi(\cdot)$ will be:

$$\pi(x_i, z) \begin{cases} > 0, & \text{if } x_{i,j} < z_j, \quad \forall j = 1, 2, \ldots, k, \\ = 0, & \text{otherwise,} \end{cases} .$$

This diversity of opinions springs from the fact that poverty is not an objective concept. Rather, it is a complex notion, the normative analysis of which may be facilitated by adopting an axiomatic approach. Thus, the properties of $F(\cdot)$ and $\pi(\cdot)$ will depend on the axioms that the poverty measures have to respect. Some axioms having been developed in the literature on multidimensional poverty measures are new, but others are simply generalizations of those inherent in the construction of one-dimensional poverty measures.

Given the difficulty of obtaining precise data on fundamental needs, we may reasonably require that a poverty measure be continuous with respect to them. [6] This circumvents the problem of small errors of measurement causing draconian changes in poverty readings. The following axiom fulfills this requirement:

Axiom 1 *Continuity: The poverty measure must not be sensitive to a marginal variation in the quantity of an attribute.*

[5] More information about this is presented in Duclos et al. (2002).

[6] See, for example, Donaldson and Weymark (1986).

Individuals' identity, or any other indicator that is irrelevant to measuring poverty, should not affect the results of the analysis. This principle is summed up in the following proposition:

Axiom 2 *Symmetry (or Anonymity): All characteristics other than the attributes used to define poverty do not impact on poverty.*

Generally, ordinal poverty comparisons occur between populations of different sizes, whence the necessity of this axiom:

Axiom 3 *The Principle of Population: If a matrix of attributes is replicated several times, global poverty remains unchanged.*

Chakravarty (1983) and Thon (1983) have introduced this axiom into poverty analysis from the income inequality literature.[7] Since the two different-sized matrix of attributes can be replicated to the same size, their poverty levels can then be directly compared.

Similarly, Egypt and Tunisia that are subject to an ordinal comparison of poverty use, for instance, different currencies. Hence, it is useful that poverty indices are independent of the units of measure. The following axiom expresses this requirement:[8]

Axiom 4 *Scale Invariance: The poverty measure is homogeneous of degree zero (0) with respect to X and z.*

This axiom will be fulfilled if any attribute is normalized by its corresponding poverty line. The individual poverty function will then have the following form:

$$\pi\left(x_i, z\right) = \pi\left(\frac{x_{i,1}}{z_1}, \ldots, \frac{x_{i,j}}{z_j}, \ldots, \frac{x_{i,k}}{z_k}\right). \tag{3}$$

Axiom 5 *Focus: The poverty measure does not change if an attribute j increases for an individual i characterized by $x_{i,j} \geq z_j$.*

[7] One of its consequences is that the poverty measure falls with increases in the size of the non-poor population.

[8] Blackorby and Donaldson (1980) distinguish this axiom from another, Transformation Invariance. This suggests that

$$P\left(X + T, z + t\right) = P\left(X, z\right).$$

We have not retained this latter axiom, because it has only been used by Tsui (2002) in a multidimensional analysis.

Using this axiom, we should find:

$$\frac{\partial \pi}{\partial x_{i,j}} = 0 \text{ if } x_{i,j} \geq z_j. \tag{4}$$

Thus, the iso-poverty curves for a poor individual run parallel to the axis of the j-th attribute when $x_{i,j} \geq z_j$.[9]

Some multidimensional poverty indices, like the HPI index, are not completely satisfying as they violate the following property:

Axiom 6 *Monotonicity: The poverty measure declines, or does not rise, following an improvement affecting any of a poor individual's attributes.*[10]

The consequence of this axiom is that iso-poverty curves are not increasing, i.e.

$$\frac{\partial \pi\left(x_i, z\right)}{\partial x_{i,j}} \leq 0 \text{ if } x_{i,j} < z_j. \tag{5}$$

Like for one-dimensional measures, it is desirable that multidimensional poverty measures be sensitive to the welfare levels of different segments of the population with homogeneous characteristics, such as age, gender, place of residence, etc. Foster and Shorroks (1991) spell out this property for a situation in which the total population can be decomposed into two subgroups (denoted respectively by a and b):

Axiom 7 *Subgroup Consistency: Let* $X \begin{bmatrix} X^a \\ X^b \end{bmatrix}$ *and* $Y \begin{bmatrix} Y^a \\ Y^b \end{bmatrix}$ *with* X^a *and* Y^a *(*X^b *and* Y^b*) being* $n^a \times k$ $\left(n^b \times k\right)$ *matrices. If* $P\left(X^a, z\right) > P(Y^a, z)$ *while* $P\left(X^b, z\right) = P\left(Y^b, z\right)$*, then*

$$P\left(X, z\right) > P\left(Y, z\right).$$

A trivial implication of the preceding axiom is that a multidimensional poverty index can be formulated as:

$$P\left(X, z\right) = F\left[\frac{1}{n}\sum_{i=1}^{n} \pi\left(x_i, z\right)\right]. \tag{6}$$

[9] An iso-poverty curve indicates the various vectors x_i that yield the same level of individual poverty, i.e. $\pi\left(x_i, z\right) = \bar{\pi}$.

[10] For example, the multidimensional poverty incidence and the HPI index may violate this axiom. Indeed, if malnutrition becomes worse among children already affected by that problem, the value of the multidimensional poverty incidence and the HPI index remain unchanged.

When $F(\cdot)$ is additive, the poverty measure $P(X,z)$ also respects the *decomposability* axiom:

Axiom 8 *Subgroup Decomposability: Global poverty is a weighted mean of poverty levels within each subgroup:*

$$P(X,z)=\frac{1}{n}\sum_{i=1}^{n}\pi(x_i,z).$$

Poverty measures that satisfy *Decomposability* enable the evaluation of each population segment's contribution to total poverty. This makes possible the conception of poverty-fighting programs that are more focussed on the most vulnerable.[11]

In addition to decomposing the population by subgroup, Chakravarty et al. (1998) also support a decomposition by attribute:

Axiom 9 *Factor Decomposability: Global poverty is a weighted mean of poverty levels by attribute.*[12]

According to Chakravarty et al. (1998), this double decomposition makes easy the design of inexpensive and efficient programs to curb poverty. It is thus particularly useful when financial constraints preclude poverty removal in an entire population segment or by a specific attribute. If the double decomposition is retained, then multidimensional poverty measures take the following form:

$$P(X,z)=\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k}\pi_j(x_{i,j},z_j). \tag{7}$$

In the event that $\pi(x_i,z_j)$ assumes of the following forms:

$$\pi(x_i,z)=\sum_{j=1}^{k}a_j\left(\frac{z_j-x_{i,j}}{z_j}\right)^{\alpha}, \tag{a}$$

we obtain a multidimensional extension of the Foster, Greer and Thorbeke (1984) poverty measures suggested by Chakravarty et al. (1998). Many others forms of

[11] More detail on the usefulness of this axiom can be found in Chakravarty et al. (1998), Tsui (2002), and Bourguignon and Chakravarty (2003).

[12] Bourguignon and Chakravarty (2003) show that, under certain conditions, a decomposition by factors necessarily arises.

$\pi\left(x_i, z_j\right)$ respecting the *factor decomposability* are also possible, like the following one:

$$\pi\left(x_i, z\right)=\sum_{j=1}^{k} a_j \ln \left[\frac{z_j}{\min \left(x_{i,j}, z_j\right)}\right], \tag{b}$$

in which case, we obtain a multidimensional extension of the Watts (1968) poverty index.

Conversely, the following multiplicative extension to the FGT class:

$$\pi(x_i, z)=\prod_{j=1}^{j}\left(\frac{z_j-x_{i,j}}{z_j}\right)^{\alpha_j}, \tag{c}$$

where α_j is a parameter reflecting poverty aversion with respect to attribute j, does not respect *decomposability by factor*. Moreover, in this case, poverty is measured across the *intersection* of various dimensions of human deprivation. In fact, an individual having the minimum required for a single attribute, but less than the minimum for all others, will not be considered part of the deprived population.

It is clear that *factor decomposability* necessarily leads to poverty measures based on the *union* of different dimensions of poverty—but the converse is not always true. For example, the index suggested by Tsui (2002), though not compatible with *factor decomposability*, is based on the *union* of the various dimensions of poverty:[13]

$$\pi(x_i, z)=\prod_{j=1}^{j}\left[\frac{z_j}{\min(x_{i,j}, z_j)}\right]^{\beta_j}-1. \tag{d}$$

Sen (1976) suggests that poverty measures should be sensitive to inequalities within the less well-off members of society. In other words, a Dalton transfer from a relatively less poor individual to a poorer one should reduce the poverty index.[14] This principle was applied by Kolm (1977) to study the problem of inequality in a multidimensional context. For a multidimensional poverty measure, Tsui (2002) introduced the following axiom:

Axiom 10 *Transfer: Poverty is not increased with matrix Y if it is obtained from matrix X by simply redistributing the attributes of the poor using a bistochastic*

[13] This is a multidimensional extension of Chakravarty's (1983) measure. Aside from decomposability by factor, this measure obeys all the axioms developed so far.

[14] Dalton (1920) observed that a transfer from a non-poor individual to a poor one improves social welfare as long as there is no reclassification of the two individuals.

transformation (and not permutation) matrix.[15]

Intuitively, the distribution reflected by matrix Y is more egalitarian than that in matrix X if extreme solutions are replaced with more mid-range ones. For example, assume two attributes such that $z_1 = 10$ and $z_2 = 12$. Let the initial distribution be characterized by $x_1\,(2,10)$ and $x_2\,(8,2)$. If Y is obtained from X using a bistochastic matrix B all of the elements of which are equal to 0.5, the two individuals will have $y_1\,(5,6)$ and $y_2\,(5,6)$, respectively. Clearly, the distribution Y is more egalitarian than X, which explains why it must contain less poverty. Thus, this property implies that the iso-poverty curves must be convex, or

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,j}} \geq 0, \quad \forall x_{i,j} < z_j. \tag{8}$$

We can confirm that the *transfer* axiom is satisfied by the Watts (1968) measure, the FGT measures when $\alpha > 1$, and the Tsui (2002) measures when $\beta_j > 0$.

There is an inequitable type of transfer that is not covered by the preceding developments. Assume always that $k = 2$, $z_1 = 10$, and $z_2 = 12$. Let $x_1\,(1,2)$, $x_2\,(5,3)$, and $x_3\,(2,7)$, and assume that after a transfer we have $y_1\,(1,2)$, $y_2\,(2,3)$, and $y_3\,(5,7)$. The correlation between the attributes increases subsequent to this transfer, i.e. an individual having more of one attribute also has more of the other attribute. Intuitively, poverty must increase, or at least not decrease, after this type of transfer. [16] The following axiom, proposed by Tsui (2002), imposes that a poverty measure should not decrease after this type of transfer:

Axiom 11 *Nondecreasing Poverty Under a Correlation Increasing Switch: Let Y be obtained from X following a series of transfers within the poor population. Let these transfers increase the correlation between attributes while no individual ceases to be poor, then*

$$P\,(Y, z) \geq P\,(X, z)\,.$$

Bourguignon and Chakravarty (2003) point out that this axiom is valid for substitutable attributes. In this situation, substitutability must be understood in terms of closeness in the nature of the attributes. In light of this, if we let, for instance, expenditure and education be two attributes with similar natures, then the

[15] The values of the elements of a doubly stochastic transformation matrix are between zero (0) and one (1). Each row (column) of such a matrix sums to one (1).

[16] Atkinson and Bourguignon (1982) suggest that a measure of social welfare must not increase after this type of transfer.

poverty of individual 3 does not decline by very much when expenditure increases, because her educational level is important. The decrease would have been greater had she been less educational attainment. It is important that the expected fall not offset the increase in poverty of individual 2, whose expenditure has decreased while his educational level is low. Analytically, when attributes are substitutable, we have

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,k}} \geq 0, \quad \forall x_{i,j} < z_j. \tag{9}$$

We ought to point out that poverty measured by twice-decomposable indices will remain unchanged subsequent to any transfer increasing the correlation between attributes. The Tsui (2002) poverty measure will necessarily increase if $\beta_j \beta_k > 0$.

However, when two attributes are deemed complementary, the fall in poverty of individual 3 must be greater, at least to the point of compensating for the increase in poverty of individual 2. The following axiom, introduced by Bourguignon and Chakravarty (2003), generalizes the preceding one:

Proposition 12 *Poverty is nondecreasing (nonincreasing) subsequent to a rise in the correlation between two attributes when these attributes are substitutes (complements).*

Analytically, when the attributes are complements, we have

$$\frac{\partial^2 \pi(x_i, z)}{\partial x_{ij} \partial x_{i,k}} \leq 0, \quad \forall x_{i,j} < z_j. \tag{10}$$

Bourguignon and Chakravarty (2003) put forward an extension to the FGT class of measures that, in addition to respecting all the axioms developed above, also allows for substitutability and complementarity among attributes: [17]

$$P_{\alpha,\gamma}(X, z) = \frac{1}{n} \sum_{i=1}^{n} \left[\left(\frac{z_1 - x_{i,1}}{z_1} \right)^{\gamma} + b^{\frac{\gamma}{\alpha}} \left(\frac{z_2 - x_{i,2}}{z_2} \right)^{\gamma} \right]^{\frac{\alpha}{\gamma}}, \tag{11}$$

where $\alpha \geq 1$, $\gamma \geq 1$, and $b > 0$. $\alpha \geq 1$ ensures that the transfer principle for a single attribute is respected for poor people. When $\alpha \geq 1$, $\gamma \geq 1$ ensures that this principle extends to individuals who are poor in two attributes simultaneously. As the value of γ increases, the iso-poverty curve becomes more convex. The

[17]To keep the presentation tractable, we use the $k = 2$ case.

elasticity of substitution between the two poverty deficits is $\frac{1}{\gamma-1}$. The (positive) magnitude of b reflects the relative weight of the second attribute vis-à-vis the first. When $\alpha \geq \gamma \geq 1$, the two attributes are substitutes and the measure given by $P_{\sigma,\gamma}(X,z)$ respects the property that *poverty is nondecreasing after an increase in correlation between the attributes*. Conversely, when $\gamma \geq \alpha$, the two attributes are complements, and $P_{\alpha,\gamma}(X,z)$ satisfies the condition that *poverty is nonincreasing subsequent to a rise in the correlation between the two attributes*. When $\gamma = 1$, the iso-poverty curves are linear for these two attributes in the case of poor individuals. Finally, as the value of γ becomes very large, the measure $P_{\alpha,\infty}(X,z)$ can be written as follows:

$$P_{\alpha,\infty}(X,z) = \frac{1}{n}\sum_{i=1}^{n}\left[1 - \min\left(1, \frac{x_{i,1}}{z_1}, \frac{x_{i,2}}{z_2}\right)\right]^{\alpha}, \qquad (12)$$

in this case, the two attributes are complementary and the iso-poverty curves assume the shape of Leontief curves.

4 Implementation to Egypt and Tunisia

Household budget surveys, which collect data on several households' attributes are required to implement the preceding measures as well as the theoretical framework behind them. In the MENA regions, only Egypt, Jordan, Morocco, Tunisia, and Palestine have conducted regular household surveys. These surveys are nationally representative, which is essential for ordinal poverty comparisons. Most of these countries, however, have not enabled satisfactory access to the collected data.

For this reason, we compare the multidimensional poverty level only between Egypt and Tunisia.[18] Micro data from the Egyptian (Tunisian) household survey for the year 1997 (1990) are being used. These are multipurpose household surveys which provide information on expenditures as well as on many other dimensions of households characteristics including education, housing, region of residence, and demographic information. The 1997 Egyptian household survey, which was done by the International Food Policy Research Institute (IFPRI), includes 2451 households: 1123 urban and 1328 rural households. In Tunisia, the

[18] Egypt and Tunisia are situated in the north of Africa with a population of almost 66.4 and 9.8 millions in 2002, respectively. The per capita GDP is 1470 current US dollars in Egypt in 2002 and 2160 current US dollars in Tunisia.

1990 household survey was conducted by the *Institut National de la Statistique* (INS). 7734 households were surveyed: 4477 urban and 3257 rural households.

For expositional simplicity, we focus on two dimensions of individual well-being, namely the total expenditures *per capita* (labelled as x_{i1}) as a proxy of monetary dimension of welfare, and the years number of schooling (labelled as x_{i2}) as a proxy of the educational attainment. The reason for choosing these two attributes is also related to data availability. For instance, there is no information on access to health services or safe water in these two surveys.

Poverty is measured at the individual level. Each individual is given the total expenditure *per capita* within the household he/she belongs to. The income poverty line, z_1, in Egypt (Tunisia) is set at 1297 Egyptian pounds (218 Tunisian dinars) for the urban area and 857 Egyptian pounds (185 Tunisian dinars) for the rural area *per capita per year*. [19] The educational poverty threshold, z_2, is defined as the end of primary school, that is, 6 years of schooling for both countries.

These country specific income poverty lines reveal that the income incidence of poverty is greater in Egypt than in Tunisia. In Tunisia, simply 7.4 percent of the population lives below the income poverty line but this ratio attains 24.4 percent in Egypt. Conversely, the education incidence of poverty appears really important in both countries, even if it is by far higher in Tunisia. Indeed, 79.3 percent of the population in Tunisia has not completed 6 years of schooling whilst the education headcount in Egypt stands at 57 percent.

Since income poverty is more important in Egypt while education poverty is higher in Tunisia, it is now instructive to use the different multidimensional poverty measures, presented in the previous section, to check whether the difference in one dimension of poverty could offset the asymmetric difference in the other dimension. [20] Table 1 in appendix presents estimates of the *intersection* bi-dimensional incidence of poverty, *union* bi-dimensional headcount ratio, as well as many others bi-dimensional poverty indices developed above. These different poverty measures are computed using the same weight for the two dimensions of poverty.

Table 1 shows that the proportion of individuals in poverty in both of the two dimensions is closely equal to the income incidence of poverty, mainly in Tunisia. This means that most of those in income poverty have not achieved pri-

[19] These poverty lines are estimated by El-Laithy et al. (1999) for Egypt and the World Bank (1995) for Tunisia using a very similar method.

[20] This issue of the trade-off between these two particular dimensions of poverty would also arise in very different contexts. For instance, designing anti-poverty policies in each country may require deciding whether it is better to reduce more income or education poverty.

mary schooling. This explains why the *intersection* bi-dimensional incidence of poverty is 2.6 times more important in Egypt than in Tunisia. Nonetheless, the higher education deprivation in Tunisia is behind the greater *union* bivariate head-count ratio in this country.

Both *intersection* and *union* bi-dimensional headcount ratio violate many of the desirable properties developed above. If we focus on *union* poverty measures that are distribution-sensitive, table 1 shows that bi-dimensional poverty are more important in Tunisia than in Egypt as long as the double decomposability is respected or the attributes are not highly substitutes. This is because low substitutability between the two attributes gives more weight for each observation to the attribute with the largest shortfall. This result remains valid when these two attributes are assumed to be, to some extents, substitutes too. Yet as the degree of substitutability rises, bi-dimensional poverty becomes more important in Egypt.

5 Robustness Analysis

In the light of the analysis conducted above, we know that some poverty measures rank Egypt and Tunisia differently to others. Ordinal poverty ranking could also be mitigated by an alternate choice of poverty lines. Thus, the stochastic dominance approach turns out to be indispensable to establish the conditions under which poverty comparisons are robust within a plausible range of poverty lines and across a pre-defined family of poverty measures. The principal findings of stochastic dominance theory in a single dimension are:[21]

Poverty decreases, or does not increase, for any possible choice of $z_j \in \left[0, z_j^*\right]$, when moving from a distribution A to a distribution B of attribute j, if the incidence of poverty under distribution A is never greater than that under distribution B. If this condition is observed, then the condition for first-order stochastic dominance holds. Otherwise, it is possible to establish a weaker condition, that of second-order stochastic dominance. This requires that poverty, as measured by the normalized poverty deficit, does not increase for any possible choice of $z_j \in \left[0, z_j^*\right]$, when moving from a distribution A to a distribution B.

While the literature dealing with issues of dominance in a one-dimensional environment (based on an axiomatic approach) is well developed, research into the multidimensional aspect is scarcely beginning, and remains an important avenue of exploration.

[21] For more information on one-dimensional robustness analysis, see Atkinson (1987) and Foster and Shorroks (1988).

Bourguignon and Chakravarty (2002) seek to establish conditions for the robustness of a given ordinal ranking, given X and z, under the assumption that the upper poverty line for each attribute remains fixed. They also assume that the poverty measure respects the *axioms of Focus, Symmetry, Principle of Population, and Subgroup decomposability*.

The distribution of attributes $x_i\left(x_{i,1}, x_{i,2}\right)$ is now replaced by the cumulative distribution function $H\left(x_1, x_2\right)$, defined on $[0, a_1] \times [0, a_2]$. The goal is to compare two distributions: H and H^*. Given the *axiom of decomposability*, poverty associated with the distribution H can be written as:

$$P(H, z) = \int_0^{a_1} \int_0^{a_2} \pi_z(x_1, x_2) dH, \tag{13}$$

where $\pi_z\left(x_1, x_2\right)$ is the level of poverty associated with an individual having attributes $\left(x_1, x_2\right)$. The poverty differential between H and H^* is given by

$$\Delta P(z) = \int_0^{a_1} \int_0^{a_2} \pi_z(x_1, x_2) d\Delta H, \tag{14}$$

where $\Delta H = H\left(x_1, x_2\right) - H^*\left(x_1, x_2\right)$. Distribution H (weakly) dominates H^* if ΔP is negative (nonpositive) for all $\pi_z\left(x_1, x_2\right)$ belonging to a given class of measures $P\left(\cdot\right)$.

Bourguignon and Chakravarty (2002) study multidimensional families of poverty measures that are in line with the *monotonicity* axiom. They distinguish between classes of measures with two substitutable, complementary, or independent attributes. They show that:

- When two attributes are substitutable, i.e. $\frac{\delta^2 \pi_x(x_1, x_2)}{\delta x_1, \delta x_2} > 0$, stochastic dominance requires first-order dominance in each dimension of poverty,

$$\Delta P(x_j) = \int_0^{x_j} d\Delta H_{u_j}(u_j) \leq 0, \quad \forall x_j \leq z_j, \tag{15}$$

  and first-order dominance across the *intersection* of the two dimensions of poverty,

$$\Delta P(x) = \int_0^{x_1} \int_0^{z_{x2}} d\Delta H(u_1, u_2), \quad \forall x_j \leq z. \tag{16}$$

- When the two attributes are complements, i.e. $\frac{\delta^2 \pi_x(x_1, x_2)}{\delta x_1, \delta x_2} < 0$, stochastic dominance also requires the first-order robustness of each dimension of poverty. Among other things, first-order dominance across the *union* of the two dimensions of poverty is required:

$$\Delta P(x) = \sum_{j=1}^{j=2} \int_0^{x_j} \Delta H_{u_j}(u_j) du_j - \int_0^{x_1} \int_0^{z_{x2}} d\Delta H(u_1, u_2) \qquad (17)$$
$$\leq 0, \quad \forall x_j \leq z_j.$$

- When the two attributes are independent: i.e. $\frac{\delta^2 \pi_x(x_1,x_2)}{\delta x_1,\delta x_2} = 0$, the selected poverty measures are twice decomposable. First-order dominance only requires the condition described by equation (15).

Figure 1 in appendix illustrates these findings where a positive difference means that there is more poverty in Egypt than in Tunisia. For Expositional simplicity, the distribution of (x_{i1}, x_{i2}) is normalized so as $x_{ij} = 100$ whenever $x_{ij} = z_j$ in each country. Hence, by plotting the cumulative percentages difference of the population below various income poverty lines (z_1) and education poverty lines (z_2), figure 1 shows that Tunisia first-order-dominates (FOD) on the income ground but Egypt FOD on the education ground. First-order dominance is then inconclusive and it is necessary to test higher order dominance to check an unambiguous ordinal ranking.

Whenever it is desirable for poverty measures to further respect the *transfer* axiom, Bourguignon and Chakravarty (2002) argue that it is hard to apply the findings of the second-order dominance analogously. In reality, this analysis requires restrictions on the signs of the second and third derivatives of the poverty function, the interpretation of which is unclear in the context of bi-dimensional poverty. [22]

Fortunately, Duclos et al. (2002) establish conditions for robustness that do not require restrictive conditions on the intervals of variation of the different z_j-s. They define the individual welfare function as:

$$\lambda(x_1, x_2) : \Re^2 \rightarrow \Re \left| \frac{\partial \lambda(x_1, x_2)}{\partial x_1} \geq 0, \ \frac{\partial \lambda(x_1, x_2)}{\partial x_2} \geq 0. \right. \qquad (18)$$

They assume that an unknown poverty frontier separates the poor from the non-poor population. This frontier is implicitly defined by $\lambda(x_1, x_2) = 0$. The set of

[22]When the chosen measures are additive over attributes and population subgroups, Bourguignon and Chakravarty (2002) show that second-order robustness simply requires second-order dominance for each attribute for all $x_j \leq z_j$.

the poor is then defined by:

$$\Lambda(\lambda) = \{(x_1, x_2) \,| \lambda(x_1, x_2) \leq 0\} \,. \tag{19}$$

Consequently, a two-dimensional poverty measure satisfying the *subgroup decomposability* axiom can be written as:

$$P(\lambda) = \int \int_{\Lambda(\lambda)} \pi(x_1, x_2, \lambda) dH(x_1, x_2), \tag{20}$$

where $\pi\left(x_1, x_2, \lambda\right)$ is the contribution of an individual characterized by the pair (x_1, x_2) to global poverty. By the *focus* axiom, this function is

$$\begin{aligned} \pi(x_1, x_2, \lambda) &\geq 0 \text{ if } \lambda(x_1, x_2) \leq 0, \\ &= 0 \text{ otherwise.} \end{aligned} \tag{21}$$

Depending on the analytical form chosen, the function $\pi\left(x_1, x_2, \lambda\right)$ measures poverty across the *intersection*, the *union*, or an intermediate combination of the two selected dimensions.

For purposes of robustness analysis, Duclos et al. (2002) consider the following multidimensional extension of the FGT class of measures:

$$P_{\alpha_1,\alpha_2}(X, z) = \int_0^{z_1} \int_0^{z_2} \left(\frac{z_1 - x_1}{z_1}\right)^{\alpha_1} \left(\frac{z_2 - x_2}{z_2}\right)^{\alpha_2} dH(x_1, x_2). \tag{22}$$

This index plays an important role in the ordinal robust comparisons of poverty, even though it measures poverty across the *intersection* of the two dimensions considered. These comparisons will be based on dominance order $r_1 = \alpha_1 + 1$ in space x_1, and $r_2 = \alpha_2 + 1$ in space x_2. $P_{0,0}\left(X, z\right)$ is the bi-dimensional incidence of poverty, i.e. the proportion of the population that is poor in both of those attributes simultaneously. $P_{1,0}\left(X, z\right)$ aggregates the x_1 poverty deficit of poor individuals with respect to the second attribute. $P_{1,1}\left(X, z\right)$ aggregates the products of the poverty deficits, normalized by the size of the population.

Rather than selecting arbitrary poverty lines and measures, Duclos et al. (2002) begin by characterizing a class of poverty measures, then specify the necessary conditions for a distribution, A, to dominate another, B, for all poverty measures belonging to the defined class. They first consider the following class of poverty measures:

$$\Pi_{1,1}(\lambda^*) = \left\{ P(\lambda) \left| \begin{array}{c} \Lambda(\lambda) \subset \Lambda(\lambda^*) \\ \pi(x, \lambda) = 0 \text{ if } \lambda(x_1, x_2) = 0 \\ \pi^{x_j} \leq 0, \quad \forall x_j \\ \pi^{x_j x_k} \geq 0, \quad \forall x_j, x_k, \end{array} \right. \right\}, \tag{23}$$

where π^{x_j} (π^{x_j,x_k}) corresponds to the first (cross) derivative of the function $\pi(x,\lambda)$ with respect to x_j (x_j and x_k). The first row of equation (23) defines the upper limit of the two poverty lines. The second indicates that poverty measures of $\Pi_{1,1}(\lambda^*)$ are continuous all along the frontier separating the poor from the non-poor segments of the population.[23] The third row stipulates that poverty measures in this class satisfy the *monotonicity* axiom. Finally, the fourth row reveals that measures in this class are compatible with the axiom underlying the substitutability of attributes.[24] Depending on the choice of functional form for $\pi(x,\lambda)$, this class may include poverty measures based on the *intersection*, the *union*, or any intermediary form of the two dimensions of poverty.

Relying on Duclos et al. (2002) framework, we can assert that poverty, as measured by any bi-dimensional index of the class $\Pi_{1,1}(\lambda^*)$ will be higher in Egypt than in Tunisia if the following condition is fulfilled:

$$\Delta P_{0,0}(x_1,x_2) > 0, \quad \forall (x_1,x_2) \in \Lambda(\lambda^*). \tag{24}$$

In other words, robustness of order $(1,1)$ requires that the percentage of the population that is poor in both attributes simultaneously be larger in Egypt, and that this holds for all ordered pairs $(z_1,z_2) \in [0,z_1^*] \times [0,z_2^*]$. Whenever this condition is fulfilled, any poverty index of class $\Pi_{1,1}(\lambda^*)$ will indicate that there is more poverty in Egypt than in Tunisia, regardless of whether this index measures poverty across the *intersection*, the *union*, or any intermediary specification.

Figure 2 in appendix illustrates the relationship of bi-dimensional poverty difference between Egypt and Tunisia to (1,1)-order dominance and the cumulative share difference of the population that is poor in both attributes. This figure clearly shows that if we admit that the income poverty line could never exceed 300 percent of the specific z_1 of each region, then we could state that poverty is unambiguously higher in Egypt than in Tunisia for all poverty yardsticks which belong to $\Pi_{1,1}(\lambda^*)$. Otherwise, the sign of $\Delta P_{0,0}(x_1,x_2)$ appears sensitive to the choice of z_j and testing for higher orders of dominance for one of the two dimensions, such as $(2,1)$ or $(1,2)$, or for both simultaneously, $(2,2)$, becomes appealing.

Hence, because it is desirable for poverty to diminish following an equalizing (Daltonian) transfer of x_1 at a given value of x_2, and that this effect is decreasing in the value of x_2, the following class of measures becomes of particular pertinence:

[23] This naturally precludes a two-dimensional incidence of poverty.

[24] Unlike Bourguignon and Chakravarty (2002), Duclos et al. (2002) reject the axiom underlying the complementarity of attributes.

$$\Pi_{2,1}(\lambda^*) = \left\{ P(\lambda) \left| \begin{array}{c} P(\lambda) \in \Pi_{1,1}(\lambda^*), \\ \pi^{x_1,x_1} \geq 0, \quad \forall x_1, \\ \pi^{x_1 x_1 x_2} \leq 0, \quad \forall x_1, x_2. \end{array} \right. \right\}. \tag{25}$$

A necessary and sufficient condition for poverty, as measured by any index of the class $\Pi_{2,1}(\lambda^*)$, to be unambiguously bigger in Egypt than in Tunisia, is that the poverty gap in x_1 for those individuals who are poor in x_2 be higher in Egypt, and that, for all the range variation of $z_j \in [0, z_j^*]$. Analytically, the condition for stochastic dominance of order $(2,1)$ requires that:

$$\Delta P_{1,0}(x_1, x_2) > 0, \quad \forall (x_1, x_2) \in \Lambda(\lambda^*). \tag{26}$$

Figure 3 in appendix provides evidence that the sign of $\Delta P_{1,0}(x_1, x_2)$ is always positive. Thus, the $(2,1)$-order stochastic dominance condition is met, no matter what the income and the education poverty line are, so that any index of $\Pi_{2,1}(\lambda^*)$ will show more deprivation in Egypt than in Tunisia. Nonetheless, the outcomes illustrated by figure 4, which is linked to the $(1,2)$-order dominance, are very similar to those provided by figure 2. This should mean that as the focus on income dimension of poverty rises, the fact that there is more multidimensional poverty in Egypt will be enhanced. This idea, nevertheless, will be moderated, mainly for a high level of income poverty line, if more attention is set on the education poverty.

6 Conclusion

Although poverty is a multifaceted issue, the literature on poverty comparisons has been largely concerned with single dimensioned indices. However, there is a clearly need among policymakers and international agencies for meaningful descriptive and normative measures of multidimensional deprivation. This paper surveys the main contributions to the literature on multidimensional poverty comparisons by means of complete and partial rankings. Poverty indices yield complete orderings because they enable to rank all pairs of multivariate distributions. Partial poverty orderings, which are linked to robustness analysis, require unanimous poverty ranking for a set of poverty lines and a class of poverty measures.

The use of these approaches is illustrated using households' data from Egypt and Tunisia. We have considered the case in which poverty is measured on two dimensions, households' expenditures *per capita* as a proxy of income deprivation and the number of schooling years of the household's head as a proxy of educational attainment. The main findings are: One-dimensional poverty is higher for education in Tunisia than in Egypt but lower for income in Tunisia. Bi-dimensional poverty is higher in Tunisia when more weight is given to education. Yet when more weight is assigned to income attribute, bi-dimensional poverty becomes more important in Egypt.

It may be thought that going beyond the monetary approach of deprivation could make the combat against poverty more complex than it already is. Indeed, multivariate poverty comparisons are not as developed as that of the monetary approach because new complexities that are inherent make difficult to find robust results, something that we have faced in the empirical implementation of this paper. Notwithstanding, the bi-dimensional picture of deprivation has enabled a better characterization of this issue and should let, on the contrary, the fight against poverty less complex. Such as, in the light of the poverty characterization in Tunisia, designing anti-poverty policy should mainly focus on the illiteracy issue rather than on the income deprivation. [25] But, in Egypt, the focus should be made on the rise of the education return to fight more and better the income deprivation.

References

[1] Adams R. H. and John Page, (2001), Holding the Line: Poverty Reduction in the Middle East and North Africa, 1970-2000. Poverty Reduction Group, The World Bank, Washington D.C.

[2] Arnand, S. et A. K. Sen (1997), Concepts of Human Development and Poverty : A Multidimensional Perspective. *Human Development Papers*, United Nations Development Programme (UNDP), New York.

[3] Atkinson, A. B. (1987), On the Measurement of Poverty. *Econometrica*, vol. 55 (4), pp. 749-764.

[4] Atkinson, A. B. and F. Bouguignon, (1982), The comparaison of Multidimensioned Distributions of Economic Status. *Review of Economic Studies*, vol. 49, 183-201.

[25]The illiteracy deprivation could also be more directly observable than the household's income, so that the problems of imperfect information and work incentives could be less acute.

[5] Blackorby, C. and D. Donaldson (1980), Ethical Indices for the Measurement of Poverty. *Econemetrica*, vol. 48, pp. 1053-1060.

[6] Bourguignon, F. and S. R. Chakravarty, (2002), Multidimensional Poverty Orderings. *DELTA*, Paris.

[7] Bourguignon, F. and S. R. Chakravarty, (2003), The Measurement of Multidimensional Poverty. *Journal of Economic Inequality*, vol. 1 (1), pp. 25-49.

[8] Chakravarty, S. R. (1983), A New Index of Poverty. *Mathematical Social Science*, vol. 6, pp. 307-313.

[9] Chakravarty, S. R., D. Mukherjee, and R. Ranade (1998), On the Family of Subgroup and Factor Decomposable Measures of Multidimensional Poverty. *Research on Economic Inequality*. vol. 8, pp. 175-194.

[10] Dalton, H. (1920), The Measurement of the Inequality of Income. *The Economic Journal*, vol. 30, pp. 348-361.

[11] Donaldson, D. and J. A. Weymark, (1986), Properties of Fixed-Population Poverty Indices. *International Economic Review*, vol. 27, pp. 667-688.

[12] Duclos, J.-Y., D. Sahn, et S. D. Younger (2002), Robust Multidimensional Poverty Comparisons.

[13] El-Laithy, H., O. El-Khawaga, and N. Riad (1999), Poverty Assessment in Egypt 1991-1996. *Economic Research Monograph*, Cairo University, Egypt.

[14] Foster, J. E., J. Greer and E. Thorbecke. (1984), A Class of Decomposable Poverty Measures. *Econometrica*, vol. 52 (3), pp. 761-765.

[15] Foster, J. E. and A. Shorroks, (1988), Poverty Orderings. *Econometrica*, vol. 56, pp. 173-176.

[16] Foster, J. E. and A. Shorroks, (1991), Subgroup Consistent Poverty Indices. *Econometrica*, vol. 59, pp. 687-709.

[17] Klasen, S. (2000), Measuring Poverty and Deprivation in South Africa. *Review of Economic and Wealth*, vol. 46 (1), pp. 33-58.

[18] Kolm, S. C. (1977), Multidimensional Egalitarism. *Quarterly Journal of Economics*, vol. 91, pp. 1-13.

[19] Sen, A. K. (1976), Poverty: An Ordinal Approach to Measurement. *Econometrica*, vol. 44 (2), pp. 219-231.

[20] Sen, A.K. (1985), *Commodities and Capabilities*. North-Holland, Amsterdam.

[21] Sen, A. K. (1992), *Inequality Reexamined*. Cambridge, MA: Havard University Press.

[22] Sen, A. K. (1999), *Development as Freedom*. Oxford: Oxford University Press.

[23] Slesnick, D. T. (1998), Empirical Approaches to the Measurement of Welfare. *Journal of Economic Literature*, vol. 36, pp. 2108-2165.

[24] Streeten, P. (1981), *First Things First: Meeting Basic Human Needs in Developing Countries*, Oxford University Press, New York.

[25] Thon, D. (1983), A poverty Measure" *Indian Economic Journal*, vol. 30, pp. 55-70.

[26] Tsui, K. (2002), Multidimensional Poverty Indices. *Social Choice and Welfare,* vol. 19, pp. 69-93.

[27] UNDP (1997), *Human Development Report*, Chap. 1, pp. 15-21 and 117-121, New York, Oxford University Press.

[28] Watts, H. (1968), An Economic Definition of Poverty. In *On Understanding Poverty*. Edited by D. P. Moynihan, New York: Basic Books.

[29] World Bank (1995) Republic of Tunisia, Poverty Alleviation: Preserving Progress while Preparing for the Future. Middle East and North Africa Region, Report N°13993-TUN.

7 Appendix

Table 1: Bi-dimensional poverty comparisons

	Egypt	Tunisia	$\Delta P(X,z)$
Intersection headcount ratio	18.5	7.2	11.4
Union headcount ratio	62.9	79.6	-16.7
Chakravarty et al. (1998), $\alpha_j = 1$	51.5	65.6	-14.1
Chakravarty et al. (1998), $\alpha_j = 2$	43.7	58.5	-14.8
Watts (1968)	260.2	349.5	-89.4
Tsui (2002), $\beta_j = 1$	26722	32139	-5417
$P_{\alpha,\gamma}(X,z), \alpha = 3, \gamma = 2,$ (Substitutes)	42.5	55.9	-13.4
$P_{\alpha,\gamma}(X,z), \alpha = 15, \gamma = 2,$ (Substitutes)	68.5	58.9	9.6
$P_{\alpha,\gamma}(X,z), \alpha = 3, \gamma = 4,$ (Complements)	40.3	55.2	-14.9
$P_{\alpha,\gamma}(X,z), \alpha = 3, \gamma = \infty,$ (Leontief)	39.9	55.1	-15.1

N.B. A positive sign of $\Delta P(X,z)$ indicates that bi-dimensional poverty is higher in Egypt than in Tunisia. Further, for the additive bi-dimensional poverty indices, the same weight, $a_j = 1$, is given to the two attributes.

Figure 1: Egypt minus Tunisia Income vs Education FOD

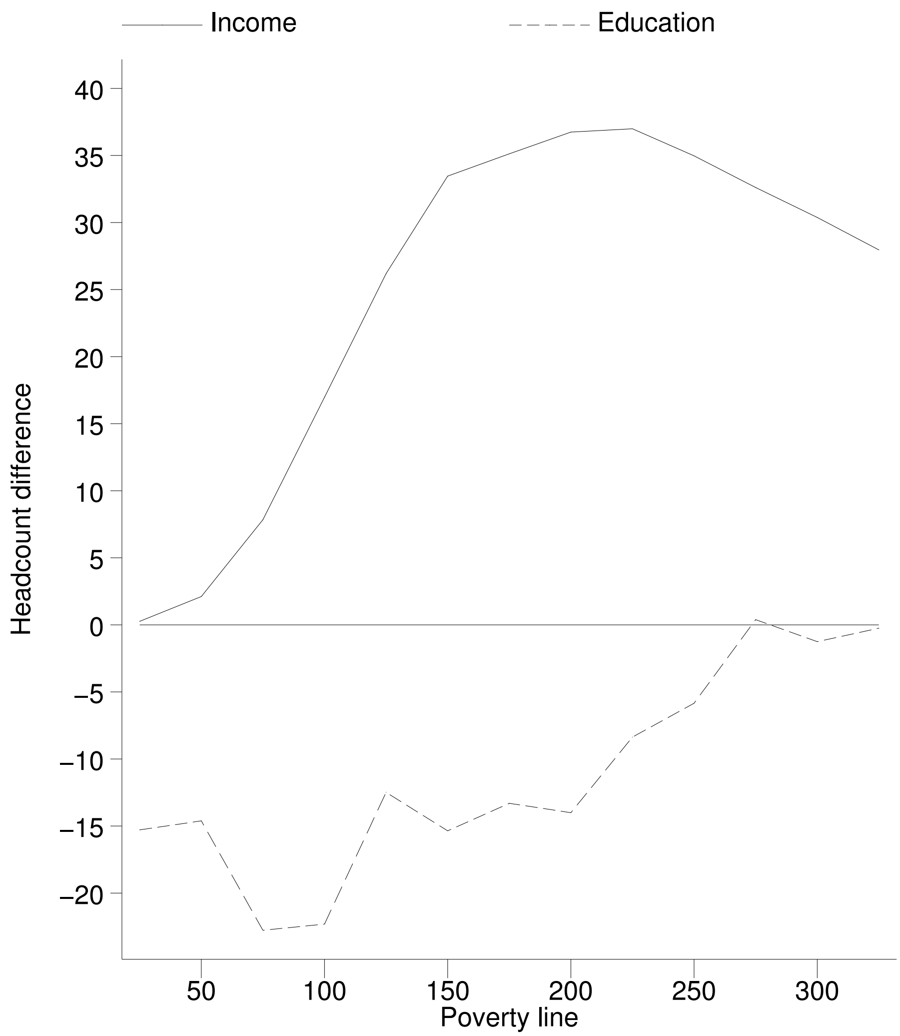

Figure 2: Egypt minus Tunisia robustness of order (1,1)

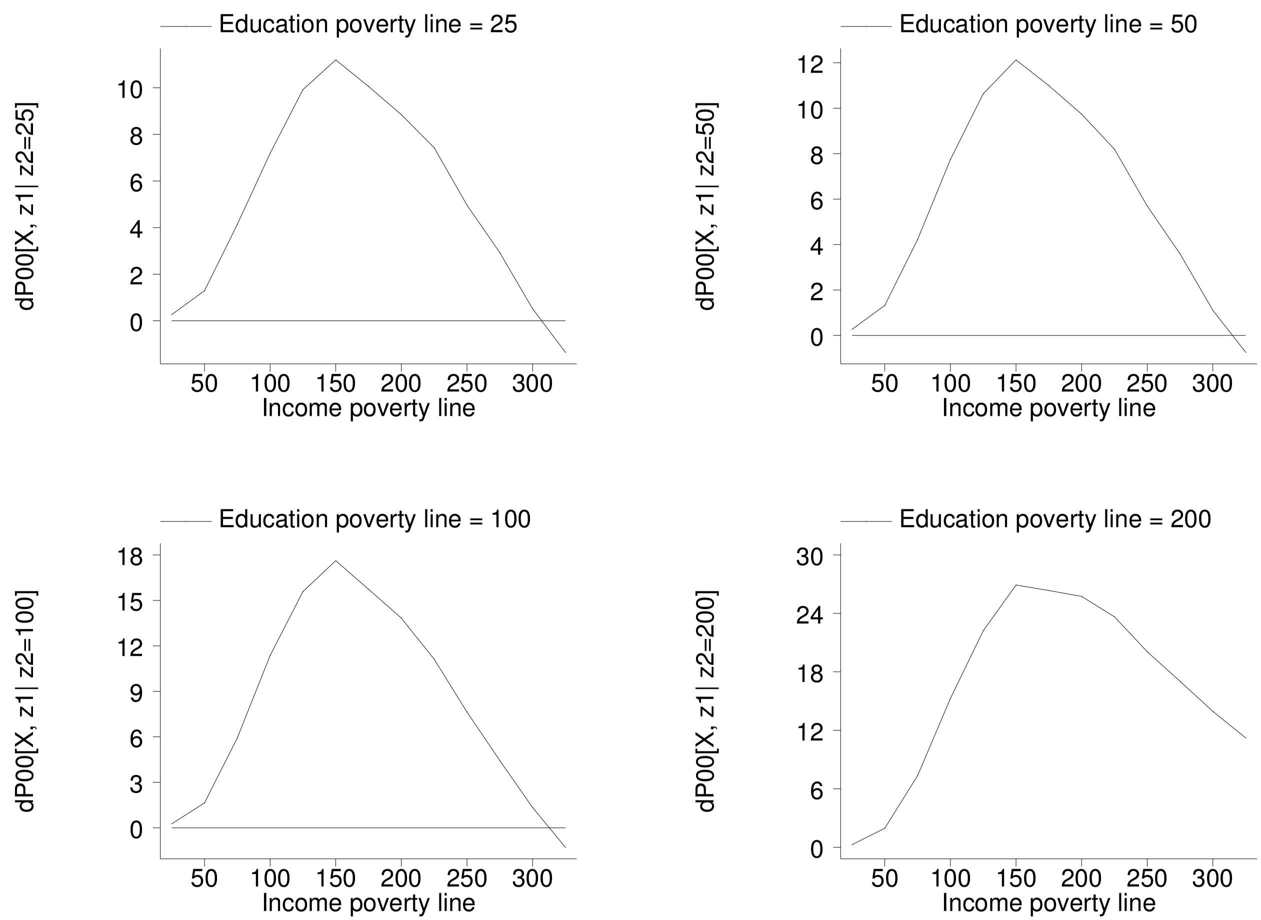

Figure 3: Egypt minus Tunisia robustness of order (2,1)

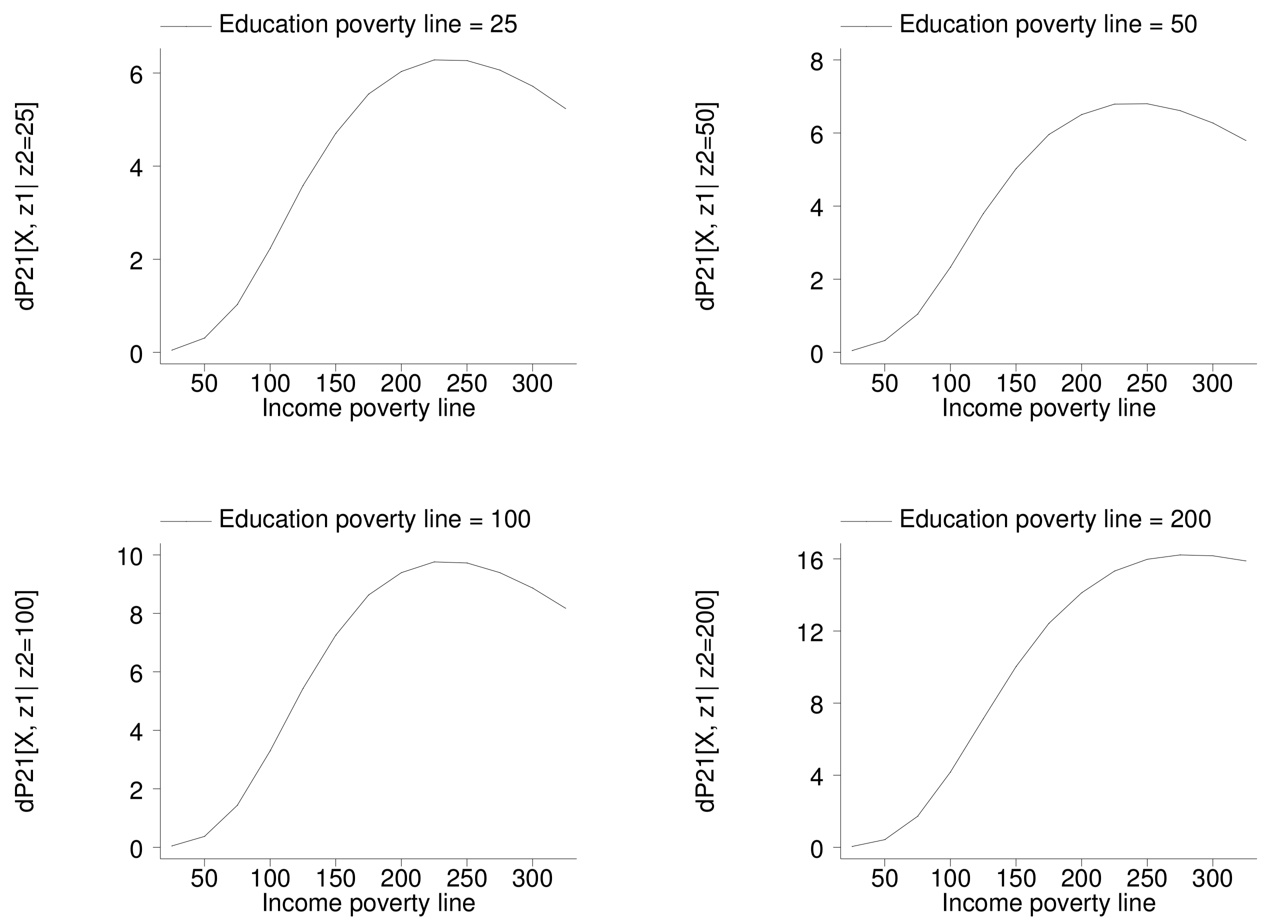

Figure 4: Egypt minus Tunisia robustness of order (1,2)