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On The Watts Multidimensional Poverty Index

A b s t r a c t

The multidimensional extension of the Watts poverty index, in addition to being subgroup and factor decomposable, satisfies a third decomposability condition which says that poverty change may be expressed as the sum of a growth component and a redistribution component. In this paper it is shown that under a set of reasonable axioms, the Watts multidimensional index is the unique index if poverty is perceived as the absolute amount of social welfare loss due to its existence. This index is relatively easy to apply and it also satisfies all basic axioms for a good index. An application of this index to world data on per capita GDP, life expectancy and literacy rates is then provided. Moreover, using the Shapley decomposition, we derive the contribution to the variation of this index between 1993 and 2002 of five determinants measuring respectively the impact of the changes in what are defined in the paper as the Watts poverty gap ratio, the Theil-Bourguignon index of inequality among the poor, the overall headcount ratio, the weights of the various dimensions and some measure of correlation between the various dimensions.

1. Introduction

Removal of poverty has been and continues to be one of the primary aims of economic policy in a large number of countries. Therefore, targeting of poverty alleviation is still a very important issue in many countries. It is thus necessary to know the dimension of poverty and the process through which it seems to be aggravated. A natural question that arises in this context is how to quantify the extent of poverty.

In a pioneering contribution, Sen (1976) regarded the poverty measurement problem as involving two exercises: (i) the identification of the poor and (ii) aggregation of the characteristics of the poor into an overall indicator that quantifies the extent of poverty. In the literature, the first problem is mostly solved by the income method, which requires specification of a poverty line representing the income required for subsistence standard of living. A person is said to be poor if his income falls below the poverty line. On the aggregation issue, Sen (1976) criticized two crude indicators of poverty, the head-count ratio (proportion of persons with incomes below the poverty line) and the income gap ratio (the difference between the poverty line and the average income of the poor, expressed as a proportion of the poverty line), because they remain unaltered under a transfer of income between two poor persons and the former also does not change if a poor person becomes poorer due to a reduction in his income. Sen (1976) also characterized axiomatically a more sophisticated index of poverty[1].

However, well-being of a population and hence its poverty, which is a manifestation of insufficient well-being, is a multidimensional phenomenon, income is only one of the many attributes on which the well-being depends. Examples of such attributes are food, housing, clothing, education, health, provision of public goods and so on. While it is true that with a higher income a person is able to improve the position of some his non-monetary attributes, it may as well be the case that markets for certain attributes do not exist, for instance, in the case of some public goods. Examples are flood

[1] Several contributions suggested alternatives and variations of the Sen index. See, for example, Takayama (1979), Blackorby and Donaldson (1980), Kakwani (1980), Clark, Hemming and Ulph (1981), Chakravarty (1983, 1983a, 1997), Foster, Greer and Thorbecke (1984), and Shorrocks (1995).

control program and malaria prevention program in an underdeveloped country. (See Ravallion, 1996; Tsui 2002 and Bourguignon and Chakravarty, 2003).

We can argue further for viewing poverty measurement problem from a multidimensional perspective. In the basic needs approach development is regarded as an improvement in the array of human needs, not just as growth of income alone (Streeten, 1981). There is a debate about the importance of low income as a determinant of under nutrition (Lipton and Ravallion, 1995). In the capability-functioning approach, where functionings deal with what a person can ultimately do and capabilities indicate a person's freedom with respect to functionings (Sen, 1985, 1992), poverty is regarded as a problem of functioning failure. Functionings here are closely approximated by attributes like literacy, life expectancy etc. An example of a multidimensional index of poverty in the functionings failure framework is the human poverty index suggested by the UNDP (1997). It aggregates the deprivations in the living standard of a population in terms of three basic dimensions of life, namely, decent living standard, educational attainment rate and life expectancy at birth.

In view of the above discussion, we assume that each person is characterized by a vector of basic need attributes and a direct method of identification of poor checks if the person has "minimally acceptable levels" (Sen, 1992 p. 139) of this set of basic needs. Therefore, the direct method considers poverty from a multidimensional perspective, more, precisely, in terms of shortfalls of attribute quantities from respective threshold levels. These threshold levels are determined independently of the attribute distributions. Since the direct method "is not based on particular assumptions of consumer behavior which may or may not be accurate", "it is supper to the income method" (Sen, 1981, p. 26). If direct information on different attributes are not available, one can adopt the income method, "so that the income method is at most a second best" (Sen,1981, p.26).

The objective of this paper is to characterize the multidimensional extension of the Watts (1968) poverty index using a social welfare approach and analyze its properties. Among various attractive properties of this index are subgroup and factor decomposability. Subgroup decomposability means that for any partitioning of the population into subgroups with respect to some homogeneous characteristic, say age, sex, race, region etc., overall poverty value becomes a weighted average of subgroup poverty values, where the weights are the population shares of the subgroups. This in turn enables us to identify the subgroups that are more afflicted by poverty and hence to design antipoverty policy. According to factor decomposability overall level of poverty is a weighted average of attribute-wise poverty levels. Clearly, the high contributing attributes require attention from policy makers for reducing their contributions in order to obtain a lower position in poverty profile. A third attractive feature of this index is that change in poverty can be decomposed into a multidimensional growth component and a multidimensional redistribution component. The growth component isolates the effect of economic growth on poverty change assuming that inequality remains unaltered; the redistribution component detects the impact of change in inequality conditional on unchanged means of attributes. The investigation of the relationship between poverty, economic growth and inequality is motivated by policy related issues such as 'trickle-down' effects of economic growth and impact of structural adjustment programs to stabilize the economy. (See, for example, Kanbur,1987,Ravallion and Huppi, 1991, Datt and Ravallion, 1992, Kakwani 1993 and Lipton and Ravallion, 1993).

The paper is organized as follows. The next section is concerned with the postulates for a multidimensional poverty index. Section 3 discusses the Watts index and presents the characterization exercise. Section 4 shows that this Watts index may be in fact expressed as a function of five determinants which are respectively what we define as the Watts poverty gap ratio, the Bourguignon(1979) -Theil(1967 index of inequality among the poor, the overall headcount ratio, the weights of the various dimensions and some measure of correlation between the various dimensions. Then, using the well-known Shapley(1953) decomposition we derive the contribution of each of these five determinants to the overall change in the Watts index. The paper ends by providing a numerical illustration based on data on the per capita GDP, life expectancy and literacy rates in various countries of the world in 1993 and 2002. Finally, section 6 concludes.

2 . Properties for an Index of Multidimensional Poverty

In this section we lay down the postulates for multidimensional poverty index. Let R^m_{++} stand for the positive orthant of the m-dimensional Euclidean space R^m. For a set of n-persons, the *ith* person possesses an m-vector $(x_{i1}, x_{i2}, ..., x_{im}) = x_i \in R^n_{++}$ of attributes. The vector x_i is the *ith* row of an $n \times m$ matrix $X \in M^n$, where M^n is the set of all $n \times m$ matrices whose entries are positive real numbers. The *jth* column $x_{.j}$ of $X \in M^n$ gives the distribution of attribute *j* (*j = 1, 2, ..., m*) among the *n* persons. Let $M = \bigcup_{n \in N} M^n$, where *N* is the set of all positive integers. For any $n \in N$, $X \in M^n$, we write n (X) (or n) for the associated population size.

In this multivariate structure a threshold is defined for each attribute. These thresholds represent the minimal quantities of the *m* attributes necessary for maintaining a subsistence level of living. Let $z = (z_1, .., z_m) \in Z$ be the vector of thresholds, where *Z* is a nonempty subset of R^m_{++}. The quantitative specification of different attributes exclude the possibility that a variable can be of qualitative type, for instance, whether a person likes or does not like his job.

In this framework, person *i* will be called poor or non-poor with respect to attribute j, equivalently, attribute *j* is meager or non-meager for person *i*, according as $x_{ij} < z_j$ or $x_{ij} \geq z_j$ and he/she is called non-poor if $x_{ij} \geq z_j$ for all *j*. Let $S_j(X)$ (or S_j) be the set of persons who are poor with respect to attribute *j* in any given $X \in M^n$,where $n \in N$ is arbitrary . As Bourguignon and Chakravarty (2003) argued a simple way of counting the number of poor here is to define the poverty indicator variable:

$$\rho(x_i; z) = 1 \text{ if } \exists\, j \in \{1, 2, .., m\} : x_{ij} < z_j,$$
$$= 0, \text{ otherwise.} \tag{1}$$

Than the number of poor in the multidimensional framework is given by:

$$n_p(X) = \sum_{i=1}^{n} \rho(x_i; z) \tag{2}$$

A multidimensional poverty index P is a non-constant real valued function defined on $M \otimes Z$. For any $X \in M, z \in Z$, the functional value $P(X; z)$ gives the extent of poverty associated with the attribute matrix *X* and the threshold vector *z*.

Sen (1976) suggested two basic postulates for an income poverty measure. These are: (i) the monotonicity axiom, which demands poverty not to decrease under a reduction in the income of a poor, and (ii) the transfer axiom, which requires that poverty should not decrease if there is a transfer of income from a poor person to anyone who is richer. Following Sen several other axioms have been suggested in the literature (see, for example, Foster, Greer and Thorbecke, 1984, Donaldson and Weymark, 1986, Cowell,1988,Chakravarty , 1990 Foster and Shorrocks, 1991 , Bourguignon and Fields, 1997 and Zheng,1997).

The properties we suggest below for an arbitrary P are immediate generalizations of different postulates proposed for an income poverty index. All properties apply for any positive integer *n*.

Focus (FOC): For any $(X;z) \in M \otimes Z$ and for any person *i* and attribute *j* such that $x_{ij} \geq z_j$, an increase in x_{ij}, given that all other attribute levels in X remain fixed, does not changes the poverty value $P(X;z)$.[2]

Normalization (NOM): For any $(X;z) \in M \otimes Z$ if $x_{ij} \geq z_j$ for all *i* and *j*, then $P(X;z)$ =0.

Monotonicity (MON): For any $(X;z) \in M \otimes Z$, any person *i* and attribute *j* such that $x_{ij} < z_j$, an increase in x_{ij}, given that other attribute levels in X remain fixed, does not increase the poverty value $P(X;z)$.

Principle of Population (POP): For any $(X;z) \in M \otimes Z$, $P(X;z) = P(X^{(k)};z)$ where $X^k = (X^1, X^2, \ldots X^k)$ with each $X^i = X$, and $k \geq 2$ is arbitrary.

Symmetry (SYM): For any $(X;z) \in M \otimes Z$, $P(X;z) = P(\pi X;z)$, where π is any permutation matrix of appropriate order.[3]

Subgroup Decomposability (SUD): For any $X^1, X^2, \ldots, X^k \in M$ and $z \in Z, P(X;z) = \sum_{i=1}^{k} \frac{n_i}{n} P(X^i;z)$, where $X = (X^1, \ldots, X^k) \in M$, n_i is the population size associated with X^i and $\sum_{i=1}^{k} n_i = n$.

Continuity (CON) : $P(X;z)$ is continuous in $(X; z)$.

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[2] One may think of a stronger version of this axiom where the condition $x_{ij} \geq z_j$ would apply simultaneously to all j. See Bourguignon and Chakravarty (2003).

Transfers Principle (TRP): For any $z \in Z$, and X, Y of the same dimension, if $X^P = BY^P$ and BP^P is not a permutation of the rows of Y^P, where $X^P(Y^P)$ is the attribute matrix of the poor corresponding to X (Y) and $B = (b_{ij})$ is some bistochastic matrix of appropriate order $\left(b_{ij} \geq 0, \sum_i b_{ij} = \sum_j b_{ij} = 1\right)$, then $P(X;z) \leq P(Y;z)$.

Non-decreasingness in Subsistence Levels of Attributes (NDS): For any $X \in M$, $P(X;z)$ is non-decreasing in z_j for all *j*.

Non-poverty Growth (NPG): For any $(X;z) \in M \otimes Z$, if Y is obtained from X by adding a rich person to the society, then $P(Y;z) \leq P(X;z)$.

Scale Invariance (SCI): For all $(X^1;z^1) \in M \otimes \mathrm{Z}$, $P(X^1;z^1) = P(X^2;z^2)$, where $X^2 = X^1 \Omega$, $z^2 = z^1 \Omega$ and $\Omega = diag\ (\lambda_1, \lambda_2, \ldots \lambda_2), \lambda_i > 0$ for all *i*.

FOC states that if a person is not poor with respect to an attribute, then giving him more of this attribute does not change the intensity of poverty, even if he/she is poor in the other attribute. Thus, FOC rules out trade off between the two attributes of a person who is poor with respect to one but non-poor with respect to the other. Thus, if education and composite good are two attributes, more education above the threshold is of no use if the composite good is below its threshold. This, however, does not exclude the possibility of a trade off if both the attributes are meager for a person. NOM is a cardinality property of the poverty index. It says that if all persons in a society are non-poor, then the index value is zero. According to MON, poverty does not increase if the condition of a poor improves. Under POP, if an attribute matrix is replicated several times, then poverty remains unchanged. Since by replication we can transform two different sized matrices into the same size, POP is helpful for inter-temporal and interregional poverty comparisons. SYM demands anonymity. Any characteristic other than the attributes under consideration, for instance, the names of the individuals, is immaterial for poverty measurement. CON ensures that minor changes in attribute and threshold quantities will not give rise to an abrupt jump in the value of the poverty index. Therefore, a continuous poverty index will not be oversensitive to minor observational errors on basic need and threshold quantities.

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[3] A square matrix of any order with entries 0 and 1 is called a permutation matrix if each of its rows and

SUD says that if a population is divided into several subgroups, say *k*, defined along ethnic, geographical or other lines, then the overall poverty is the population share weighted average of subgroup poverty levels. The contribution of subgroup *i* to overall poverty is $n_i P(X^i; z)/n$ and overall poverty will precisely fall by this amount if poverty in subgroup *i* is eliminated. $(n_i P(X^i; Z)/\ nP(X; Z))100$ is the percentage contribution of subgroup *i* to total poverty. Each of these statistics is useful to policymakers because they become helpful for isolating subgroups of the population that are more susceptible to poverty (see Anand, 1997; Chakravarty, 1983, Foster, Greer and Thorbecke, 1984 and Foster and Shorrocks, 1991). Using SUD we can write the poverty index as

$$P(X; z) = \frac{1}{n} \sum_{i=1}^{n} p\,(x_i; z). \qquad (1)$$

Since $p\,(x_i; z)$ depends only on person *i's* attributes, we call it, 'individual poverty function'. TRP shows that if we transform the attribute matrix Y^P of the poor in *Y* to the corresponding matrix X^P in *X* by some equalizing operation, then poverty under *X* will not be higher than that under *Y*.

Between two identical communities, the one with higher subsistence levels of one or more basic needs should not have a lower poverty because of higher deprivation of the poor resulting from increased subsistence quantities. This is what NDS demands. According to NPG poverty should not increase if a rich person joins the society. Thus, under FOC, NPG says that the poverty index is a non-increasing function of the population size (see Kundu and Smith, 1983, Subramanian, 2002 and Chakravarty, Kanbur and Mukherjee, 2005). Finally, SCI means that the poverty index should be invariant under scale transformations of attribute and threshold levels. In other words, deprivation resulting from poverty is viewed in terms of proportionate shortfalls of attribute quantities from respective threshold values.

We will now consider a property which takes care of the essence of multidimensional measurement through correlation between attributes. By taking into account the association of attributes, as captured by the degree of correlation between them, this property also underlines the difference between single and multidimensional

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columns sums to one.

poverty measurements. To illustrate the property, consider the two-person two-attribute case, where both the attributes are meager for these persons. Suppose that $x_{11} > x_{21}$ and $x_{12} < x_{22}$. Now consider a switch of attribute 2 between the two persons. This switch increases the correlation between the attributes because person 1 who had more of attribute 1 has now more of attribute 2 too and that is why we refer to it as a correlation increasing switch between two poor persons. Next, suppose that attributes 1 and 2 are substitutes, or, in other words, that one attribute may compensate for the lack of another in the definition of individual poverty. Then increasing the correlation between the attributes will not decrease poverty. Indeed, the switch just defined does not modify the marginal distribution of each attribute but reduces the extent to which the lack of one attribute may be compensated by the availability of the other. An analogous argument will establish that poverty should not increase under a correlation increasing switch if the two attributes are complements.

We state this principle formally for substitutes as:

Non-decreasing Poverty Under Correlation Increasing Switch (NDP): For any $(X:z) \in M \otimes Z$, if $Y \in M$ is obtained from X by a correlation increasing switch between two poor persons, then $P(X;z) \leq P(Y;z)$ if the two attributes are substitutes.

The corresponding property which demands poverty not to increase under such a switch when the attributes are complements is denoted by NIP. If a poverty index does not change under a correlation increasing switch, then it treats the attributes as 'independents[4].

3. The Multidimensional Watts Index and Its Characterization

[4] For further discussions on this issue, see Atkinson and Bourguignon (1982) and Bourguignon and Chakravarty (1999, 2003). Bourguignon and Chakravarty (1999) employed this property to examine the elasticity of substitution between proportional shortfalls of attributes from respective thresholds.

The multidimensional Watts poverty index is defined by

$$P_W(X;Z) = \frac{1}{n}\sum_{j=1}^{m} \sum_{i \in S_j} \delta_j \log \frac{z_j}{x_{ij}} \tag{2}$$

where $n \in N$ and (X; z) $\in M^n \otimes Z$ are arbitrary, $\delta_j \geq 0$ with some inequalities being strict. It is easy to see that the log linear index P_W satisfies FOC, NOM, MON, POP, SYM, SUD, CON, NDS, NPG, TRP and SCI. A rank preserving transfer of some quantity of an attribute from a poor person to a poorer poor reduces P_W by a larger amount the poorer the recipient is. That is, P_W attaches greater weight to transfers lower down the attribute scale. Since P_W remains unchanged under a correlation increasing switch, it regards the attributes as 'independents'. The multiplicative factor δ_j is a scale parameter in the sense that given (X; z) an increase in δ_j for any j increases P_W.

Let X^1 and X^2 be the attribute matrices of a society in periods 1 and 2 and the corresponding population sizes be n_1 and n_2 respectively. That is, $X^1 \in M^{n_1}$ and $X^2 \in M^{n_2}$. Then assuming that the threshold quantities are fixed, the poverty change in this society between the two periods is

$$\begin{aligned}\Delta P &= P_W\ (X^2;z) - P_W(X^1;z) \\ &= \left[H(X^2)\left(\frac{1}{n_p(X^2)}\sum_{j=1}^{m}\sum_{i\in S_j(X^2)} \delta_j \log\frac{\eta_j^2}{x_{ij}^2}\right) - H(X^1)\left(\frac{1}{n_p(X^1)}\sum_{j=1}^{m}\sum_{i\in S_j(X^2)} \delta_j \log\frac{\eta_j^1}{x_{ij}^1}\right)\right] \\ &+ \left[H(X^2)\sum_{j=1}^{m}\delta_j \log\frac{z_j}{\eta_j^2} - H(X^1)\sum_{j=1}^{m}\delta_j \log\frac{z_j}{\eta_j^1}\right] \end{aligned} \tag{3}$$

where $\mathrm{H}(X^k) = n_P(X^k)/n(X^k)$ and η_j^k are respectively the multidimensional head count ratio and mean of the quantities of attribute *j* possessed by the poor in period *k* (*k*= 1, 2). The first third bracketed term on the right hand side of (3) is the redistribution component which shows the change in poverty due to a change in inequality of the poor, keeping the means of their attribute quantities constant at a reference level. The inequality index that appears here, shown in the first brackets of the term, is the multidimensional extension of Bourguignon(1979)-Theil's(1967) mean logarithmic deviation. The other term of the expression, the growth component, shows the change in the poverty index due to changes in means of attribute quantities of the poor while holding their inequality constant at a reference level.

We may note that the scale factor δ_j may be assumed to reflect the importance we attach to attribute j in our aggregation. We define the Watts poverty index for attribute j as

$$P_W\left(x_{\cdot j};z_j\right)=\delta_j \sum_{i\in S_j} \frac{z_j}{x_{ij}} , \qquad (4)$$

where $(X;\ z) \in M \otimes Z$ is arbitrary. Thus, δ_j may also be interpreted to reflect the importance that the government attaches for alleviating poverty for attribute j. The percentage contribution of attribute j to total poverty is $\left(P_W(x_{\cdot j};z_j)/P_W(X;z\ \right)100$. The elimination of poverty for attribute j will lower community poverty precisely by the amount $P_W\left(x_{\cdot j};z_j\right)$.

Having discussed different properties of the Watts index, we can now characterize it axiomatically. Since poverty is inversely related to well-being, for a particular person the extent of poverty can be interpreted as the disutility due to being poor. For an income based poverty index, Chakravarty (1983) and Hagenaars (1987) interpreted poverty as the fraction of welfare losses due to existence of poverty using utilitarian and Gini type social welfare functions. In contrast, Zheng (1993) regarded it as absolute amount of welfare loss. In this paper we take a similar approach.

Definition: For any arbitrary $n \in N$, $(X;z)\in M^n \otimes$ Z, a poverty index is defined as

$$P(X;Z)=W((\mathrm{z,z,......,z})')-W(\hat{X}) \qquad (5)$$

where $\hat{X}$ is the censored attribute matrix corresponding to X, that is, $\hat{x}_{ij} = \min\left\{x_{ij}, z_j\right\}$, W is any real valued social welfare function defined on the set of all censored attribute matrices, and prime ‘ / ’ denotes transpose. Thus, P is the size of welfare loss that results due to shortfall of attribute quantities of poor persons from the respective thresholds. At this stage we do not impose any restriction on W. Note that by definition, P satisfies FOC and NOM.

We can now present the following theorem.

Theorem 1: The only poverty index of the form (5) that satisfies CON, SUD, MON and SCI is the multidimensional Watts index given by (2).

Proof: By repeated application of SUD, we can write any poverty index $P(X;z)$ as $\frac{1}{n}\sum_{i=1}^{n} p(x_i;z)$, where $X \in M^n$ and p is the individual poverty function. This in turn shows that the poverty index given by (5) must be of the form

$$P(X;Z) = \frac{1}{n}\sum_{i=1}^{n}\left[f(z) - f(\hat{x}_i)\right], \tag{6}$$

where $f : R_{++}^m \to R^1$. By CON, f is continuous and MON demands that f is non-decreasing.

Let there be k persons having q and t and $(n-k)$ persons having u and v such that

$$\begin{aligned} & n\, f(z) - k\, f(\hat{q}) - (n-k)\, f(\hat{u}) \\ &= n\, f(z) - k\, f(\hat{t}) - (n-k)\, f(\hat{v}) \end{aligned} \tag{7}$$

That is, the poverty level for the censored attribute matrix where k persons have the vector $\hat{q}$ and $(n-k)$ persons have the vector $\hat{v}$ is same as that corresponding to the censored attribute matrix in which k persons have the vector $\hat{t}$ and (n-k) persons have the vector $\hat{v}$. We rewrite (7) as

$$k\, f(\hat{q}) + (n-k)\, f(\hat{u}) = k\, f(\hat{t}) + (n-k) f(\hat{v}) \tag{8}$$

from which it follows that

$$\frac{f(\hat{q}) - f(\hat{t})}{f(\hat{u}) - f(\hat{v})} = -\frac{n-k}{k}. \tag{9}$$

By SCI we have $nf(z) - kf(\hat{q}) - (n-k) f(\hat{u}) = nf(z\Omega) - kf(\hat{q}\Omega) - (n-k) f(\hat{u}\Omega)$, where $\Omega =$ $diag(\lambda_1, \ldots, \lambda_m)$, $\lambda_i > 0$ for all i. Likewise, $nf(z) - kf(\hat{t}) - (n-k) f(\hat{v}) =$ $nf(z\Omega) - kf(\hat{t}\Omega) - (n-k) f(\hat{v}\Omega)$. In view of (7), it then follows that

$$kf(\hat{q}\Omega) + (n-k)\, f(\hat{u}\,\Omega) = k\, f(\hat{t}\Omega) + (n-k) f(\hat{v}\,\Omega) \tag{10}$$

from which we get

$$\frac{f(\hat{q}\,\Omega) - f(\hat{t}\,\Omega)}{f(\hat{u}\,\Omega) - f(\hat{v}\,\Omega)} = -\frac{n-k}{k}. \tag{11}$$

Combining (9) and (11) we get

$$\frac{f(\hat{q}\,\Omega) - f(\hat{t}\,\Omega)}{f(\hat{q}) - f(\hat{t})} = \frac{f(\hat{u}\,\Omega) - f(\hat{v}\,\Omega)}{f(\hat{u}) - f(\hat{v})} \tag{12}$$

Because of continuity of *f,* without loss of generality the above ratios can be assumed to be rational numbers. It is clear from (12) that

$$\frac{f(\hat{q}\,\Omega) - f(\hat{t}\Omega)}{f(\hat{q}) - f(\hat{t})} = A\ (\Omega) \tag{13}$$

Assuming that $\hat{t}$ is fixed, we can rewrite (13) as

$$f(\hat{q}\Omega) = A(\Omega) f(\hat{q}) + B(\Omega) \tag{14}$$

Non-decreasingness of *f* requires that A $(\Omega) \geq 0$.

The solutions to the functional equation (14) are given by

$$\alpha + \beta \prod_{j=1}^{m} (\hat{q}_j)^{\delta_j} \tag{15a}$$

and

$$\alpha + \sum_{j=1}^{m} \delta_j \log \hat{q}_j, \tag{15b}$$

where α is an arbitrary constant, β and δ_j have to chosen appropriately so that different postulates for a poverty index are satisfied (see Acze′l, Roberts and Rosenbaum, 1986).

Substituting the functional form (15a) in (6), we note that the resulting index does not fulfil SCI. Hence the form of f given by (15a) is ruled out. Substitution of (15b) in (6) shows that the corresponding poverty index is the Watts index. MON, that is, non-decreasingness of f demands that $\delta_j \geq 0$. Non-constancy of the poverty index shows that some of the inequalities $\delta_j \geq 0$ will be strict.

This establishes the necessity part of the theorem. The sufficiency is easy to verify. □

The general poverty index in (6) includes many indices like the one that corresponds to the multidimensional Gini welfare function. SUD excludes all such non-additive welfare functions. CON and MON further restrict the class of welfare functions or poverty indices. Finally, scale invariance picks up the Watts index of poverty as the unique index. Note that for TRP to hold we need quasi-concavity of *f* (Kolm, 1977), which is clearly satisfied.

Tsui (2002) developed a joint characterization of the multidimensional Watts index and a multidimensional extension of the Chakravarty (1983) index using subgroup consistency as an axiom, where subgroup consistency demands that reduction in poverty of any subgroup will lead to a reduction of the national poverty. Clearly, all subgroup decomposable poverty indices are subgroup consistent. Another important difference between the two characterizations is that while we regard poverty as the absolute size of welfare loss, the approach adopted by Tsui (2002) is non-welfarist.

4. Decomposing the Variations in the Watts Multidimensional Poverty Index

The objective of this section is two-fold. We first determine analytically the formula for the change in the Watts index between two periods, which in turn will be translated into the Shapley(1953) decomposition for poverty change.

4.1. Deriving the Expression for the Change in the Watts Multidimensional Poverty Index

The unidimensional Watts poverty index is defined as

$$P_{WU} = (1/n)\sum_{i=1}^{n_p} \log(\gamma / s_i) \tag{16}$$

where n is the total number of individuals, n_p is the number of poor, γ is the (income)poverty line and s_i is the income of individual i.

This index may also be written as

$$P_{WU} = (n_p / n)[\sum_{i=1}^{n_p} (1/n_p)\log(\gamma / s_p) + \sum_{i=1}^{n_p} (1/n_p)\log(s_p / s_i)] \tag{17}$$

where s_p is the mean income of the n_p poor individuals.

Let us now recall that the Bourguignon (1979)-Theil (1967) index of income inequality can be expressed as

$$L = \log s_a - (1/n)\sum_{i=1}^{n} \log s_i \tag{18}$$

where s_a is the arithmetic mean of the incomes.

Therefore the Bourguignon-Theil index of income inequality among the poor (L_p) will be

$$L_p = \log s_p - (1/n_p)\sum_{i=1}^{n_p} \log s_i \tag{19}$$

Now, the income gap ratio poverty index, *IGR*, is defined as

$$IGR = \sum_{i=1}^{n_p} (\gamma - s_i)/(n_p\gamma) = (1/n_p)[n_p - \sum_{i=1}^{n_p} (s_i/\gamma)] = 1 - (s_p/\gamma) \tag{20}$$

A close look at the first expression in brackets on the right hand side of (17) shows that it is conceptually similar to the income gap ratio since it can be written in difference form under logarithmic transformation and it will be zero when the incomes of all the poor individuals are identical to the poverty line. We could call this index the "Watts poverty gap ratio" and denote it by $P_{W,PGR}$,that is,

$$P_{W,PGR} = \sum_{i=1}^{n_p} (1/n_p)\log(\gamma/s_p) = \log(\gamma/s_p) \tag{21}$$

One may observe that $P_{W,PGR}$ corresponds more or less to the percentage gap between the poverty line γ and the mean income of the poor s_p.

Combining expressions (17), (19) and (21) we finally have

$$P_{WU} = H(P_{W,PGR} + L_P) \tag{22}$$

where, as defined in section 3,*H* denotes the headcount ratio (n_p/n).

Expression (17) may be extended to the multidimensional case so that the multidimensional Watts poverty index $P_W(X;z)$, under the assumption that $\delta_j = 1$ for all j, will be

$$\begin{aligned} P_W(X;z) = (n_p/n)\{&\sum_{i=1}^{m}(n_{pj}/n_p)[\sum_{j=1}^{n_{pj}}(1/n_{pj})\log(z_j/x_{pj})] \\ &+ \sum_{i=1}^{m}(n_{pj}/n_p)[\sum_{j=1}^{n_{pj}}(1/n_{pj})\log(x_{pj}/\eta_j)]\} \end{aligned} \tag{23}$$

where ,as stated in section 3, η_j is the mean value of indicator j among those individuals who are considered as poor with respect to indicator j and n_{pj} (more precisely, $n_{pj}(X)$) the number of individuals who are poor with respect to indicator j in the attribute matrix X.

It is clear that expression (23) may also be expressed as a multidimensional generalization of (22) and written as

$$P_W(X;z) = H[\sum_{j=1}^{m}(n_{pj}/n_p)(P_{W,PGRj} + L_{pj})] \tag{24}$$

with

$$P_{W,PGRj} = \sum_{i=1}^{n_{pj}}(1/n_{pj})\log(z_j/s_{pj}) \tag{25}$$

and

$$L_{pj} = \sum_{i=1}^{n_{pj}}(1/n_{pj})\log(\eta_j/x_{ij}) \tag{26}$$

Note that $P_{W,PGR,j}$ may be called the "Watts poverty gap ratio" for indicator j while L_{pj} represents the Bourguignon-Theil index of inequality for indicator j ,defined in section 3.

It should be stressed that the sum over all indicators j of all the n_{pj}'s may be greater than n_p (X), the number of poor in X. This is why we need to introduce a normalizing factor ω expressed as $\omega = (\sum_j n_{pj}/n_p)$ and then we rewrite (24) as

$$P_W(X;z) = H[(\sum_{j=1}^{m}n_{pj}/n_p)\sum_{j=1}^{m}(n_{pj}/\sum_j n_{pj})(P_{W,PGRj} + L_{pj})] \tag{27}$$

or, in short, as

$$P_W(X;\gamma) = H[\omega\sum_{j=1}^{m}\sigma_j(P_{W,PGRj} + L_{pj})] \tag{28}$$

with $\sigma_j = (n_{pj}/\sum_j n_{pj})$.

Let us now add subscripts 1 and 0 to refer to the period in which multidimensional poverty is measured. We assume that the threshold quantities remain fixed. The change ΔP_W between the values of the Watts multidimensional index at times 0 and 1 will then be expressed as

$$\Delta P_W = \{H_1[\omega_1\sum_{j=1}^{m}\sigma_{j1}(P_{W,PGRj1} + L_{pj1})]\} - \{H_0[\omega_0\sum_{j=1}^{m}\sigma_{j0}(P_{W,PGRj0} + L_{pj0})]\} \tag{29}$$

$$\Leftrightarrow \Delta P_{W} = f(\Delta H, \Delta\omega, \Delta\sigma_{j}, \Delta P_{W,PGRj}, \Delta L_{pj}) \tag{30}$$

where the operator Δ refers to the variation between times 0 and 1 of the five variables that appear in (30). It is then not difficult to apply the Shapley decomposition to (29) to derive the contribution to the overall variation in poverty (in the Watts multidimensional index) of the change in the headcount ratio *H*, in the parameter ω (an indicator of the degree of intersection of the various sets of poor, that is, of the correlation between the different poverty dimensions), in the weights $\sigma_j = (n_{pj} / \sum_j n_{pj})$, in each of the "Watts poverty " indicators $P_{W,PGR,j}$ and in each of the Bourguignon-Theil measures of the inequality among the poor L_{pj}.

4.2. On the Concept of Shapley Decomposition

The concept of Shapley (1953) decomposition is a technique borrowed from game theory but extended to applied economics by Shorrocks (1999) and Sastre and Trannoy (2002). Let us explain it briefly.

Assume an indicator *I* is a function of three determinants *a,b,c* and is written as *I*= *I*(*a,b,c*). *I* could be an index of inequality but more generally any function of variables, this function being linear or not.

There are obviously 3!=6 ways of ordering these three determinants *a, b* and *c*:

(a,b,c),(a,c,b),(b,a,c),(b,c,a),(c,a,b),(c,b,a) (31)

Each of these three determinants may be eliminated first, second or third. The respective (marginal) contributions of the determinants *a,b,c* will hence be a function of all the possible ways in which each of these determinants may be eliminated. Let for example *C(a)* be the marginal contribution of *a* to the indicator *I(a,b,c)*.

If *a* is eliminated first its contribution to the overall value of the indicator *I* will be expressed as *I(a,b,c) – I(b,c)* ,where *I(b,c)* corresponds to the case where *a* is equal to zero. Since expression (31) indicates that there are two cases in which a appears first and may thus be eliminated first we will give a weight of (2/6) to this possibility.

If *a* is eliminated second, it implies that another determinant has been eliminated first (and been assumed to be equal to 0). Expression (31) indicates that there are two cases in which this possibility occurs, the one denoted in (31) as (*b,a,c*) and the one denoted as (*c,a,b*). In the first case the contribution of a will be written as $I(a,c) - I(c\ )$,while in the second it is expressed as $I(a,b) - I(b)$. To each of these two cases we evidently give a weight of (1/6).

Finally if *a* is eliminated third, it implies that both *b* and *c* are assumed to be equal to 0. Expression (31) indicates that there are two such cases, one is (*b,c,a*) and the other is (*c,b,a*). Since we may assume that when each of the three determinants is equal to 0, the indicator *I* is equal to 0, we may write that the contribution of a in this case will be equal to $I(a) - 0 = I(a)$ and evidently we have to give a weight of (2/6) to such a possibility since there are two such cases.

We may therefore summarize what we have just explained by stating that the marginal contribution *C(a)* of the determinant a to the overall value of the indicator *I* may be written as

$$C(a) = (2/6)[I(a,b,c) - I(b,c)] + (1/6)[I(a,c) - I(c)] + (1/6)[I(a,b) - I(b)] + (2/6)I(a) \quad (32)$$

One can similarly determine the marginal contribution *C(b)* of *b* and *C(c)* of *c* and then find out that

$$I(a,b,c) = C(a) + C(b) + C(c) \quad (33)$$

This Shapley decomposition may be also applied in a similar way to the case where one wants to understand the respective contributions to the change over time in the value of the indicator *I*, this change being written as ΔI, of the variations over time in the values of the three determinants *a, b* and *c*, these variations being expressed as Δa, Δb and Δc. Since in (30) the change in the multidimensional Watts poverty index is expressed as a funtion of five variables , we have to extend the Shapley decomposition described previously to five determinants. Let us, to simplify, call *a, b, c*, *d* and *e* the five determinants ΔH, $\Delta\eta$, $\Delta\sigma_j$, $\Delta P_{W,PGR,j}$, ΔL_{pj} of the Watts index that appear in (30) with $a = \Delta H$, $b = \Delta\omega$, $c = \Delta\sigma_j$, $d = \Delta P_{W,PGR,j}$ and $e = \Delta L_{pj}$ where *c, d* and *e* refer to the simultaneous changes in all the dimensions *j* of $\Delta\sigma_j$, $\Delta P_{W,PGR,j}$ and ΔL_{pj}.

It is then easy to show that the contribution *C(a)* of a in such a case, assuming, to simplify, that *I* refers to the multidimensional Watts index, may be expressed as

$$
\begin{aligned}
C(a) = \; & (24/120)\,[I(a,b,c,d,e) - I(b,c,d,e)] \\
& + (6/120)[I(a,c,d,e) - I(c,d,e)] \\
& + (6/120)\,[I(a,b,d,e) - I(b,d,e)] \\
& + (6/120)\,[I(a,b,c,e) - I(b,c,e)] \\
& + (6/120)\,[I(a,b,c,d) - I(b,c,d)] \\
& + (4/120)\,[I(a,b,c) - I(b,c)] \\
& + (4/120)\,[I(a,b,d) - I(b,d)] \\
& + (4/120)\,[I(a,b,e) - I(b,e)] \\
& + (4/120)\,[I(a,c,d) - I(c,d)] \\
& + (4/120)\,[I(a,c,e) - I(c,e)] \\
& + (4/120)\,[I(a,d,e) - I(d,e)] \\
& + (6/120)\,[I(a,b) - I(b)] \\
& + (6/120)\,[I(a,c) - I(c)] \\
& + (6/120)\,[I(a,d) - I(d)] \\
& + (6/120)\,[I(a,e) - I(e)] \\
& + (24/120)\,I(a)
\end{aligned}
\tag{34}
$$

One can similarly compute the marginal contributions of *b, c, d* and *e* and here also the sum of the contributions of *a, b, c, d* and *e* is equal to the value of the indicator *I*. To understand well (34) it should be remembered that when a determinant appears in one of the lines of (34) it means that it is different from 0 whereas if it does not appear it means it is equal to 0. Thus, for example, the line before the last one which is expressed as (6/120) [*I(a,e) – I(e)*] should be translated as

$$
\begin{aligned}
& (6/120)[\Delta P_{WM}(\Delta H \neq 0; \Delta\omega = 0; \Delta\sigma_j = 0; \Delta P_{W,PGRj} = 0; \Delta L_{pj} \neq 0) \\
& \quad - \Delta P_{WM}(\Delta H = 0; \Delta\omega = 0; \Delta\sigma_j = 0; \Delta P_{W,PGRj} = 0; \Delta L_{pj} \neq 0)]
\end{aligned}
\tag{35}
$$

Note that in (35) ΔH=0 means that when computing the change in the value of the Watts index one has to assume that the headcount ratio did not change between times 0 and 1 whereas when it is written that $\Delta H \neq 0$ it implies that we have assume that it changed. Similar interpretations hold concerning the changes $\Delta\omega$, $\Delta\sigma_j$, $\Delta P_{W,PGR,j}$ and ΔL_{pj}.

5. The Empirical Results

We have applied this decomposition technique to data on the per capita GDP, life expectancy and literacy rates of the countries for which the figures were available (164 countries representing a population of 5.3469 billions of individuals in 1992 and 5.9980 in 2002). The data were obtained from the World Development Reports for the years 1994 and 2003 (see the Appendix for information on the data used).

As is well-known these three variables are the main elements determining the Human Development Index HDI which is computed every year by the World Development Programme. The index HDI depends also on school enrollment rates but we have not taken this variable into account in order to maximize the number of countries for which the data were available. For each of the three dimensions that we selected , we had to determine a “poverty line”. For life expectancy we decided that any country in which life expectancy was smaller than 60 years should be considered as a “poor country” from the point of view of that dimension. Similarly whenever the literacy rate in a country was smaller than 60% that country was “labeled” poor as far as the literacy dimension is concerned. Finally for the per capita GDP we did not adopt the 1$ or 2$ a day criterion ,which is often adopted by international agencies. We rather decided that any country in which the per capita GDP was smaller than 5$ day should be classified as poor from the point of view of income (per capita GDP). This corresponds to an annual per capita GDP of $1825.

Table 1 gives some basic information on the poverty rates by dimension as well as on the overall poverty rate which corresponds to the “union” of the individuals considered as poor on the various dimensions. Note that all the computations are “population-weighted” which means that the weight of each country corresponds to its weight in the overall population and that if a country’s indicator is below the poverty line defined previously for each dimension, each individual in this country will be assumed to be poor. It thus appears that 45 countries were poor in both 1993 and 2002 according to the life expectancy dimension. This corresponded to 12.8% of the “World Population” in

1993 and 12.3% in 2002. As far as the literacy rate is concerned, there were 40 poor countries in 1993 and 28 in 2002, the corresponding shares of the "World Population" being respectively 30.2% and 13.0%. This sharp decrease is in great part due to the fact that in 2002 India was no more considered as poor according to that dimension. Finally, for the dimension given by the Per Capita GDP we observe that 44 countries were poor in 1993 and 38 in 2002. The corresponding population shares were 30.2% and 13.0%. Note that for this dimension too India ceased to be poor in 2002.

Table 2 gives some information on the value of the indicators used to compute the Watts multidimensional poverty index. Thus we observe that the weights given to the various dimensions are not equal and varied quite a lot between 1993 and 2002. We recall that the weight σ_j is equal to the ratio of the number of the poor computed on the basis of dimension j and the sum of the number of poor computed on the basis of the different dimensions. It appears that in 1993 the percentage of poor on the basis of the per capita GDP was much smaller than that computed on the basis of the two other dimensions since the weight σ_j corresponding to the per capita GDP was equal to 17.2% , while the weights corresponding to life expectancy and the literacy rates were respectively equal to 42.2% and 40.6%. In 2002, on the contrary, the three weights were almost identical, being respectively equal to 33.3% (per capita GDP), 31.4% (life expectancy) and 35.3% (literacy rate).

As far as the Watts poverty gap ratios are concerned we may observe that it is in fact approximately equal to the percentage difference between the poverty line for the indicator under review and the mean value of this indicator among those considered as poor. We thus observe that in 1993 this percentage gap was equal to 14.2% for the per capita GDP, 25.3% for the life expectancy and 43.9% for the literacy rate. All these gaps increased between 1993 and 2002 though not proportionately. Thus in 2002 the percentage gap between the poverty line and the average value of the indicator among the poor was equal to 20.6% for the per capita GDP, to 31.6% for the life expectancy and to 44.0% for the literacy rate.

For the Theil-Bourguignon inequality index among the poor, which can be considered as giving the gap in percent between the arithmetic and geometric mean of the distribution among the poor of the indicator corresponding to each dimension, we observe a greater degree of inequality, in both years, for the literacy rates than for the other two dimensions. Thus in 1993 (2002) this percentage gap was 0.3% (1.0%) for the per capita GDP , 1.8% (2.8%)for the life expectancy and 3.5% (6.1%) for the literacy rate.

Let us finally take a look at the value of the normalizing coefficient k whose definition was given just after equation (28). This coefficient is equal to the ratio of the *sum* over the *"union"* of the number of poor computed on the basis of the three dimensions. It should be clear that the greater the correlation between the individuals classified as poor according to the different dimensions, the smaller ω. Since we observe in table 2 that this coefficient decreased between 1993 and 2002 from 2.065 to 1.877, we may conclude that this correlation increased during that period of 9 years, meaning that those countries found to be poor according to one dimension are more likely in 2002 than in 1993 to be classified as poor according to another dimension.

Table 3 finally gives the value of the contribution of the various determinants of the multidimensional Watts poverty index to the overall variation in this index observed between 1993 and 2002, these contributions having been computed on the basis of the so-called Shapley decomposition (see section 4.2). We observe that world poverty, computed on the basis of the three dimensions given by the per capita GDP, life expectancy and literacy rate, decreased by close to 50% between 1993 and 2002 (from 0.247 to 0.131). The whole change was in fact a consequence of the decrease in the overall headcount ratio (see, Table 1). The contribution of the other determinants (normalization coefficient ω, the weights σ of the poverty dimensions, the Watts poverty gap ratio and the Theil-Bourguignon index of inequality among the poor) were quite small and cancelled out (see, table 3).

6.**Concluding Remarks**

Using different postulates for an indicator of mutidimensional poverty, we have developed a welfare theoretic characterization of the Watts index of mutidimensional poverty and investigated its properties from different perspectives. We show that the change in poverty between two periods , as demonstrated by this index, can be neatly broken down into growth and equity components, and also into Shapley decomposition components. An empirical illustration of the index has been provided using per capita GDP, life expectancy and literacy rate data for several countries for the periods 1993 and 2002.

Table 1: Basic Data on Poverty by Dimension

Indicator	1993	2002
Total number of countries	164	164
Total "World Population"	5346.9	5998.0
Number of poor countries by life expectancy	45	45
Number of poor countries by literacy	40	28
Number of poor countries by per capita GDP	44	38
Poor population by life expectancy	684.0	735.9
Poor population by literacy	1682.2	692.8
Poor population by per capita GDP	1616.1	778.2
Share of poor population in total "World Population" according to life expectancy dimension	12.8%	12.3%
Share of poor population in total "World Population" according to literacy dimension	31.5%	11.6%
Share of poor population in total "World Population" according to per capita GDP dimension	30.2%	13.0%
Share of poor population according to the three criteria together ("union" of the separate poverty rates for each of the three dimensions)	36.1%	19.6%

Table 2: Value in 1993 and 2002 of the Determinants of the Multidimensional Watts Poverty Index, broken down by Poverty Dimension

Year 1993

Headcount Ratio H_0	Coefficient ω_0	Poverty Dimension	Weight of the Poverty Dimension σ_0	"Watts Poverty Gap Ratio" for the Dimension P_{IGR0}	Theil-Bourguignon Index of Inequality Among the Poor for the Dimension L_{P0}
0.36062	2.06529	**Per Capita GDP**	0.17176	0.14197	0.00313
		Life Expectancy	0.42242	0.25263	0.01838
		Literacy Rate	0.40582	0.43930	0.03461

Year 2002

Headcount Ratio H_1	Coefficient ω_1	Poverty Dimension	Weight of the Poverty Dimension σ_1	"Watts Poverty Gap Ratio" for the Dimension P_{IGR1}	Theil-Bourguignon Index of Inequality Among the Poor for the Dimension L_{P1}
0.19602	1.87709	**Per Capita GDP**	0.33345	0.20608	0.00959
		Life Expectancy	0.31392	0.31571	0.02845
		Literacy Rate	0.35262	0.44018	0.06130

Table 3: Results of the Shapley Decomposition of the Poverty Change between 1993 and 2002

Value of the Watts Multi-Dimensional Poverty Index		Variation between 1993 and 2002 in the Value of the Watts Multi-dimensional Poverty Index	Contribution of the Various Components to the Overall Change in the Watts Multidimensional Poverty Index				
In 1993	**In 2002**	ΔP_W	**Contribution of the Headcount Ratio *H***	**Contribution of the coefficient ω**	**Contribution of the weights σ of the poverty dimensions**	**Contribution of the "Watts Poverty Gap Ratio" P_{IGR}**	**Contribution of the Theil-Bourguignon Index of Inequality among the poor L_P**
0.24707	0.13128	-0.11579	-0.11153	-0.01795	0.01668-	0.02180	0.00857

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Appendix: Data on per Capita GDP, Life Expectancy and Literacy Rate by Country in 1993 and 2002

	Country	1993				2002			
		Life exp.	Liter. rate	Per Capita GDP	Population (thousands)	Life exp.	Liter. rate	Per Capita GDP	Population (thousands)
1	Albania	72.0	85.0	2200	3.4	73.6	98.7	4830	3.1
2	Algeria	67.3	58.8	5570	26.7	69.5	68.9	5760	31.3
3	Angola	46.8	42.5	674	10.3	40.1	42.0	2130	13.2
4	Antigua and Barbuda	74.0	96.0	5369	0.1	73.9	85.8	10920	0.1
5	Argentina	72.2	96.0	8350	33.8	74.1	97.0	10880	38.0
6	Armenia	72.8	98.8	2040	3.5	72.3	99.4	3120	3.1
7	Australia	77.8	99.0	18530	17.6	79.1	99.0	28260	19.5
8	Austria	76.3	99.0	19115	7.9	78.5	99.0	29220	8.1
9	Azerbaijan	70.7	96.3	2190	7.4	72.1	97.0	3210	8.3
10	Bahamas	73.2	98.1	16180	0.3	67.1	95.5	17280	0.3
11	Bahrain	71.7	84.1	15500	0.5	73.9	88.5	17170	0.7
12	Bangladesh	55.9	37.0	1290	115.2	61.1	41.1	1700	143.8
13	Barbados	75.7	97.1	10570	0.3	77.1	99.7	15290	0.3
14	Belarus	69.7	97.9	4244	10.2	69.9	99.7	5520	9.9
15	Belgium	76.5	99.0	19540	10.0	78.7	99.0	27570	10.3
16	Belize	73.7	70.0	4610	0.2	71.5	76.9	6080	0.3
17	Benin	47.8	34.3	1650	5.1	50.7	39.8	1070	6.6
18	Bolivia	59.7	81.5	2510	7.1	63.7	86.7	2460	8.6
19	Botswana	65.2	68.0	5220	1.4	41.4	78.9	8170	1.8
20	Brazil	66.5	82.4	5500	156.5	68.0	86.4	7770	176.3
21	Brunei Darussalam	74.3	87.0	18414	0.3	76.2	93.9	19210	0.3
22	Bulgaria	71.2	93.0	4320	8.9	70.9	98.6	7130	8.0
23	Burkina Faso	47.5	18.0	780	9.8	45.8	12.8	1100	12.6
24	Burundi	50.3	33.7	670	6.0	40.8	50.4	630	6.6
25	Cambodia	51.9	35.0	1250	9.7	57.4	69.4	2060	13.8
26	Cameroon	56.3	60.8	2220	12.5	46.8	67.9	2000	15.7
27	Canada	77.5	99.0	20950	28.8	79.3	99.0	29480	31.3
28	Cape Verde	64.9	68.1	1820	0.4	70.0	75.7	5000	0.5
29	Central Afr. Rep.	49.5	56.0	1050	3.2	39.8	48.6	1170	3.8
30	Chad	47.7	46.0	690	6.0	44.7	45.8	1020	8.3
31	Chile	73.9	94.7	8900	13.8	76.0	95.7	9820	15.6
32	China	68.6	80.0	2330	1196.4	70.9	90.9	4580	1294.9
33	Colombia	69.4	90.6	5790	34.0	72.1	92.1	6370	43.5
34	Comoros	56.2	56.2	1130	0.6	60.6	56.2	1690	0.7
35	Costa Rica	76.4	94.5	5680	3.3	78.0	95.8	8840	4.1
36	Cote d'Ivoire	50.9	37.8	1620	13.3	41.2	49.7	1520	16.4
37	Cuba	75.4	95.2	3000	10.9	76.7	96.9	5259	11.3

38	Cyprus	77.1	94.0	14060	0.7	78.2	96.8	18360	0.8
39	Czech Republic	71.3	99.0	8430	10.3	75.3	99.0	15780	10.2
40	Denmark	75.3	99.0	20200	5.2	76.6	99.0	30940	5.4
41	Djibouti	48.4	44.2	775	0.6	45.8	65.5	1990	0.7
42	Dominican Republic	69.7	81.2	3690	7.5	66.7	84.4	6640	8.6
43	Dominica	72.0	94.0	3810	0.1	73.1	76.4	5640	0.1
44	Ecuador	69.0	89.0	4400	11.0	70.7	91.0	3580	12.8
45	Egypt	63.9	49.8	3800	60.3	68.6	55.6	3810	70.5
46	El Salvador	66.8	70.4	2360	5.5	70.6	79.7	4890	6.4
47	Equatorial Guinea	48.2	76.4	1800	0.4	49.1	84.2	30130	0.5
48	Estonia	69.2	99.0	3610	1.6	71.6	99.8	12260	1.3
49	Ethiopia	47.8	33.6	420	51.9	45.5	41.5	780	69.0
50	Fiji	71.6	90.6	5530	0.8	69.6	92.9	5440	0.8
51	Finland	75.8	99.0	16320	5.1	77.9	99.0	26190	5.2
52	France	77.0	99.0	19140	57.6	78.9	99.0	26920	59.8
53	Gabon	53.7	60.3	3861	1.2	56.6	71.0	6590	1.3
54	Gambia	45.2	36.6	1190	1.0	53.9	37.8	1690	1.4
55	Georgia	72.9	94.9	1750	5.4	73.5	100.0	2260	5.2
56	Germany	76.1	99.0	18840	80.9	78.2	99.0	27100	82.4
57	Ghana	56.2	62.0	2000	16.4	57.8	73.8	2130	20.5
58	Greece	77.7	93.8	8950	10.4	78.2	97.3	18720	11.0
59	Grenada	71.0	98.0	3118	0.1	65.3	94.4	7280	0.1
60	Guatemala	65.1	54.6	3400	10.0	65.7	69.9	4080	12.0
61	Guinea Bissau	43.7	52.8	860	1.0	45.2	39.6	710	1.4
62	Guinea	44.7	33.9	1800	6.3	48.9	41.0	2100	8.4
63	Guyana	65.4	97.7	2140	0.8	63.2	96.5	4260	0.8
64	Haiti	56.8	43.4	1050	6.9	49.4	51.9	1610	8.2
65	Honduras	67.9	71.4	2100	5.3	68.8	80.0	2600	6.8
66	Hong Kong	78.7	91.5	21560	5.8	79.9	93.5	26910	7.0
67	Hungary	69.0	99.0	6059	10.2	71.7	99.3	13400	9.9
68	Iceland	78.2	99.0	18640	0.3	79.7	99.0	29750	0.3
69	India	60.7	50.6	1240	901.5	63.7	61.3	2670	1049.5
70	Indonesia	63.0	82.9	3270	191.7	66.6	87.9	3230	217.1
71	Iran	67.7	66.1	5380	64.2	70.1	77.1	6690	68.1
72	Ireland	75.4	99.0	15120	3.5	76.9	99.0	36360	3.9
73	Israel	76.6	95.0	15130	5.3	79.1	95.3	19530	6.3
74	Italy	77.6	97.4	18160	57.1	78.7	98.5	26430	57.5
75	Jamaica	73.7	84.1	3180	2.4	75.6	87.6	3980	2.6
76	Japan	79.6	99.0	20660	124.5	81.5	99.0	26940	127.5
77	Jordan	68.1	84.8	4380	4.9	70.9	90.9	4220	5.3
78	Kazakhstan	69.7	97.5	3710	17.0	66.2	99.4	5870	15.5
79	Kenya	55.5	75.7	1400	26.4	45.2	84.3	1020	31.5
80	Kuwait	75.0	77.4	21630	1.8	76.5	82.9	16240	2.4
81	Kyrgyzstan	69.2	97.0	2320	4.6	68.4	97.0	1620	5.1
82	Lao	51.3	54.6	1458	4.6	54.3	66.4	1720	5.5
83	Latvia	69.0	99.0	5010	2.6	70.9	99.7	9210	2.3

84	Lebanon	68.7	91.7	2500	2.8	73.5	86.5	4360	3.6
85	Lesotho	60.8	69.5	980	1.9	36.3	81.4	2420	1.8
86	Libya	63.4	73.7	6125	5.0	72.6	81.7	7570	5.4
87	Lithuania	70.3	98.4	3110	3.7	72.5	99.6	10320	3.5
88	Luxembourg	75.8	99.0	25390	0.4	78.3	99.0	61190	0.4
89	Madagascar	56.8	45.8	700	13.9	53.4	67.3	740	16.9
90	Malawi	45.5	54.7	710	10.5	37.8	61.8	580	11.9
91	Malaysia	70.9	82.2	8360	19.2	73.0	88.7	9120	24.0
92	Maldives	62.4	92.8	2200	0.2	67.2	97.2	4798	0.3
93	Mali	46.2	28.4	530	10.1	48.5	19.0	930	12.6
94	Malta	76.2	87.0	11570	0.4	78.3	92.6	17640	0.4
95	Mauritania	51.7	36.7	1610	2.2	52.3	41.2	2220	2.8
96	Mauritius	70.4	81.7	12510	1.1	71.9	84.3	10810	1.2
97	Mexico	71.0	89.0	7010	90.0	73.3	90.5	8970	102.0
98	Moldova Rep.	67.6	96.4	2370	4.4	68.8	99.0	1470	4.3
99	Mongolia	63.9	81.7	2090	2.3	63.7	97.8	1710	2.6
100	Morocco	63.6	41.7	3270	25.9	68.5	50.7	3810	30.1
101	Mozambique	46.4	37.9	640	15.1	38.5	46.5	1050	18.5
102	Myanmar	57.9	82.4	650	44.6	57.2	85.3	1027	48.9
103	Namibia	59.1	40.0	3710	1.5	45.3	83.3	6210	2.0
104	Nepal	53.8	26.3	1000	20.8	59.6	44.0	1370	24.6
105	Netherlands	77.5	99.0	171340	15.3	78.3	99.0	29100	16.1
106	New Zealand	75.6	99.0	16720	3.5	78.2	99.0	21740	3.8
107	Nicaragua	67.1	65.0	2280	4.1	69.4	76.7	2470	5.3
108	Nigeria	50.6	54.1	1540	105.3	51.6	66.8	860	120.9
109	Niger	46.7	12.8	790	8.6	46.0	17.1	800	11.5
110	Norway	77.0	99.0	20370	4.3	78.9	99.0	36600	4.5
111	Oman	69.8	35.0	10420	2.0	72.3	74.4	13340	2.8
112	Pakistan	61.8	36.4	2160	132.9	60.8	41.5	1940	149.9
113	Panama	72.9	90.0	5890	2.5	74.6	92.3	6170	3.1
114	Papua New Guinea	56.0	70.5	2530	4.1	57.4	64.6	2270	5.6
115	Paraguay	70.1	91.5	3340	4.7	70.7	91.6	4610	5.7
116	Peru	66.3	87.8	3320	22.9	69.7	85.0	5010	26.8
117	Philippines	66.5	94.2	2590	64.8	69.8	92.6	4170	78.6
118	Poland	71.1	99.0	4702	38.3	73.8	99.7	10560	38.6
119	Portugal	74.7	86.2	10720	9.8	76.1	92.5	18280	10.0
120	Qatar	70.6	78.5	22910	0.5	72.0	84.2	19844	0.6
121	Romania	69.9	96.9	3727	23.0	70.5	97.3	6560	22.4
122	Russian Federation	67.4	98.7	4760	147.8	66.7	99.6	8230	144.1
123	Rwanda	47.2	58.0	740	7.6	38.9	69.2	1270	8.3
124	Saint Lucia	72.0	82.0	3795	0.1	72.4	94.8	5300	0.1
125	Samoa Western	67.8	98.0	3000	0.2	69.8	98.7	5600	0.2
126	Sao Tome Principe	67.0	60.0	600	0.1	69.7	83.1	1317	0.2
127	Saudi Arabia	69.9	61.3	12600	17.1	72.1	77.9	12650	23.5
128	Senegal	49.5	31.4	1710	7.9	52.7	39.3	1580	9.9

129	Seychelles	71.0	88.0	4960	0.1	72.7	91.9	18232	0.1
130	Sierra Leone	39.2	29.6	860	4.3	34.3	36.0	520	4.8
131	Singapore	74.9	90.3	19350	2.8	78.0	92.5	24040	4.2
132	Slovakia	70.9	99.0	5620	5.3	73.6	99.7	12840	5.4
133	Solomon Islands	70.5	62.0	2266	0.4	69.0	76.6	1590	0.5
134	South Africa	63.2	81.0	3127	39.7	48.8	86.0	10070	44.8
135	Spain	77.7	98.0	13660	39.5	79.2	97.7	21460	41.0
136	Sri Lanka	72.0	89.6	3030	17.9	72.5	92.1	3570	18.9
137	St Vincent	71.0	91.0	3552	0.1	74.0	83.1	5460	0.1
138	Sudan	53.2	43.8	1350	26.6	55.5	59.9	1820	32.9
139	Suriname	70.5	92.5	3670	0.4	71.0	94.0	6590	0.4
140	Swaziland	57.8	74.9	2940	0.8	35.7	80.9	4550	1.1
141	Sweden	78.3	99.0	17900	8.7	80.0	99.0	26050	8.9
142	Switzerland	78.1	99.0	22720	7.1	79.1	99.0	30010	7.2
143	Syrian	67.3	68.7	4196	13.7	71.7	82.9	3620	17.4
144	Tajikistan	70.4	96.7	1380	5.8	68.6	99.5	980	6.2
145	Tanzania Rep of	52.1	65.5	630	28.0	43.5	77.1	580	36.3
146	Thailand	69.2	93.6	6350	57.6	69.1	92.6	7010	62.2
147	Togo	55.2	49.2	1020	3.9	49.9	59.6	1480	4.8
148	Trinidad and Tobago	71.7	97.6	8670	1.3	71.4	98.5	9430	1.3
149	Tunisia	68.0	64.1	4950	8.6	72.7	73.2	6760	9.7
150	Turkey	66.7	81.1	4210	59.6	70.4	86.5	6390	70.3
151	Turkmenistan	65.1	97.7	3128	3.9	66.9	98.8	4300	4.8
152	USA	76.1	99.0	24680	257.9	77.0	99.0	35750	291.0
153	Uganda	44.7	59.7	910	19.9	45.7	68.9	1390	25.0
154	Ukraine	69.3	95.0	3250	51.6	69.5	99.6	4870	48.9
155	United Arab Emirates	73.9	78.2	20940	1.8	74.6	77.3	22420	2.9
156	United Kingdom	76.3	99.0	17230	57.9	78.1	99.0	26150	59.1
157	Uruguay	72.6	97.0	6550	3.1	75.2	97.7	7830	3.4
158	Uzbekistan	69.4	97.2	2510	21.9	69.5	99.3	1670	25.7
159	Vanuatu	65.4	65.0	2500	0.2	68.6	34.0	2890	0.2
160	Venezuela	71.8	90.6	8360	20.9	73.6	93.1	5380	25.2
161	Viet Nam	65.5	92.5	1040	71.3	69.0	90.3	2300	80.3
162	Yemen	50.4	41.1	1600	13.2	59.8	49.0	870	19.3
163	Zambia	48.6	76.2	1110	8.9	32.7	79.9	840	10.7
164	Zimbabwe	53.4	84.0	2100	10.7	33.9	90.0	2400	12.8
	Total				5346.9				5998.0

Source: World Development Reports for the years 1994 and 2003.

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