Robust Multidimensional Poverty Comparisons

by

Jean-Yves Duclos
Department of Economics and CIRPÉE, Université Laval, Canada,

David Sahn
Food and Nutrition Policy Program, Cornell University

and

Stephen D. Younger
Food and Nutrition Policy Program, Cornell University

Introduction

- It is common to assert that poverty is multi-dimensional, yet most empirical poverty work is one dimensional
- When more than one indicator of well-being is used, poverty comparisons are either made for each indicator independently of the others, or are performed using an aggregation of the multiple indicators into a single index.

But:

- aggregation of indicators can be arbitrary, relying on some normative or statistical assumptions
- aggregation across individuals of individual poverty statuses requires a poverty index – no such index has been devised that has received unanimous approval
- multidimensional poverty comparisons also require estimation of multidimensional poverty lines: an ethically and empirically problematic procedure.

- Purpose of paper: to determine whether *truly* multidimensional poverty comparisons can be made robust

  1. to the aggregation of multiple indicators,
  2. to the selection of multidimensional poverty lines,
  3. to the use of multidimensional poverty indices,
  4. and to the presence of sampling variability in the estimators used.

- We need to make an important distinction between *intersection* and *union* definitions of poverty.

- We also ask: ”What is the range of poverty lines in all dimensions over which we can be sure that poverty is lower for A than for B?”

Multiple indicators of well-being

Notation

- Let x and y be two indicators of individual well-being (for instance, income, expenditures, caloric consumption, life expectancy, height, body mass, the extent of personal safety and freedom)
- Denote by

$$\lambda(x, y) : \Re^2 \rightarrow \Re \left| \frac{\partial \lambda(x, y)}{\partial x} \geq 0, \frac{\partial \lambda(x, y)}{\partial y} \geq 0 \right. \quad (1)$$

  a summary indicator of individual well-being (analogous to utility).

1. Poverty frontier defined implicitly by the locus $\lambda(x, y) = 0$ (analogous to the usual downward-sloping indifference curves). See **Figure** 1.
2. The set of the poor is then obtained as:

$$\Lambda(\lambda) = \{(x, y) \,|(\lambda(x, y) \leq 0\} . \tag{2}$$

- Let the joint distribution function be $F(x, y)$.

- For analytical simplicity, we focus on classes of additive multidimensional poverty indices, $P(z_x(y), z_y)$:

$$P(\lambda) \;=\; \int\int_{\Lambda(\lambda)} \pi(x, y; \lambda)\, dF(x, y), \qquad (3)$$

  where $\pi(x, y; \lambda)$ is the contribution to poverty of an individual with well-being indicators x and y

- Depending on the shape of the function $\lambda(x, y)$, this allows for a mixture of both an *intersection* and a *union* approach to measuring multidimensional poverty. See **Figure** 1.

- Bi-dimensional extension of the FGT (Foster, Greer, and Thorbecke (1984)) index:

$$P^{\alpha_x,\alpha_y}(z_x, z_y) = \int_0^{z_y} \int_0^{z_x} (z_x - x)^{\alpha_x} (z_y - y)^{\alpha_y} \, dF(x, y) \tag{4}$$

  See **Figure** 2 for an example.

- Denote by π^x the first derivative of $\pi(x, y; \lambda)$ with respect to x, and so on.

- Then define the following class $\ddot{\Pi}^{1,1}(\lambda^*)$ of bidimensional poverty indices:

$$\ddot{\Pi}^{1,1}(\lambda^*) = \left\{ P(\lambda) \left| \begin{array}{l} \Lambda(\lambda) \subset \Lambda(\lambda^*) \\ \pi(x,y;\lambda) = 0, \text{ whenever } \lambda(x,y) = 0 \\ \pi^x \leq 0 \text{ and } \pi^y \leq 0 \; \forall x, y \\ \pi^{xy} \geq 0, \; \forall x, y. \end{array} \right. \right\} \quad (5$$

The last line on the right of (5) is the only debatable assumption:

Justification:

1. a ”substitutability” assumption: the more someone has of x, the less is overall poverty deemed to be reduced if his value of y is increased.
2. non-decreasing poverty under a ”correlation-increasing switch”: consider Figure 8.

- Denote by $\Delta F = F_A - F_B$ the difference between a function F for A and for B. We then have:

  Theorem 1 ($\ddot{\Pi}^{1,1}$ poverty dominance)

$$\Delta P(\lambda) > 0,\ \forall P(\lambda) \in \ddot{\Pi}^{1,1}(\lambda^*), \qquad (6)$$

$$\textit{iff}\ \Delta P^{0,0}(x,y) > 0,\ \forall (x,y) \in \Lambda(\lambda^*). \qquad (7)$$

  See Figure 1 again.

Higher order dominance tests

1. For higher-order dominance: we either increase the order in one dimension or in both simultaneously.

2. Either approach adds further assumptions on the effects of changes in either x or y on aggregate poverty – and thus limits the applicable class of poverty measures.

3. These further assumptions impose that indices react increasingly favorably to increases in living standards at the bottom of the distribution of well-being.

To illustrate this:

$$\ddot{\Pi}^{2,1}(\lambda^*) = \left\{ P(\lambda) \middle| \begin{array}{l} P(\lambda) \in \ddot{\Pi}^{1,1}(\lambda^*) \\ \pi^x(x,y;\lambda) = 0 \text{ whenever } \lambda(x,y) = 0 \\ \pi^{xx}(x,y;\lambda) \geq 0 \; \forall x, \\ \text{and } \pi^{xxy}(x,y;\lambda) \leq 0, \; \forall x,y. \end{array} \right\} \quad (8)$$

This leads to the following dominance condition:

Theorem 2 ($\ddot{\Pi}^{2,1}$ poverty dominance)

$$\begin{array}{c} \Delta P(\lambda) > 0, \; \forall P(\lambda) \in \ddot{\Pi}^{2,1}(\lambda^*) \\ \textit{iff } \Delta P^{1,0}(x,y) > 0, \; \forall (x,y) \in \Lambda(\lambda^*). \end{array} \quad (9)$$

Relevance of the methods

The methods are more general than two other common ones:

- One approach has been to combine many indicators of well-being into one, unidimensional index, and then compare that index across populations. The best-known example is the Human Development Index (UNDP, 1990).
  - Choosing to compare a single aggregate welfare index essentially reduces the domain for the test to a single line emanating from the origin and being closer to the x or y axis according to the weight that x and y receive in the welfare index.

- A second approach is to compare many indicators of well-being independently: *i.e.* looking at the univariate dominance curve for each dimension of well-being.
  - It is possible that the univariate dominance curve for A lies above that for B, but that A is not above B at one or more interior points in the test domain shown in **Figure** 1.
    - Importance of capturing ”multiple” poverty
  - It is possible for the univariate dominance surfaces to cross but for A's surface to be above B's for a large area of interior points in the test domain. Consider **Figure** 3.

Examples

1. Are rural people poorer than the urban ones in Viet Nam?

    - People living in rural areas tend to be poorer when judged by expenditures or income alone.
    - However: possible that people are better nourished in rural than urban areas, *ceteris paribus*, because they have tastes for foods that provide nutrients at a lower cost, or because unit prices of comparable food commodities are lower.

- To test this, we measure welfare in two dimensions: *per capita* household expenditures and nutritional status, as measured by a child's gender and age standardized height, transformed into standard deviation or z-scores. (Use 1993 Viet Nam Living Standards Measurement Survey.)

- Results shown in **Figure** 4 for $s_x = s_y = 1$.
  - y axis measures the height-for-age z-score (stunting)
  - x axis measures the *per capita* expenditures for the child's household
  - z axis measures the cumulative proportion of children that fall below the points defined in the (x, y) domain.
- We test for a significant difference in the dominance surface at each point of a grid, and reject the null of non-dominance of A by B only if all of the test statistics have the right sign and are significantly different from 0.

- **Figure** 4 indicates clearly that, over almost the entire range of expenditures and stunting, rural children are poorer than urban.
- **Table** 1 shows whether these statements are statistically significantly at the 5% level: a negative sign indicates that the urban dominance surface is significantly below the rural one
- The conclusion that rural children are poorer than urban ones is valid for almost any intersection, union or intermediate poverty frontier.

2. Second example tests for first-order poverty dominance in three dimensions: ”Did poverty decline in Ghana between 1993 and 1998?”

   - Three welfare variables for children under five years old: survival probability, height-for-age z-score (stunting), and index of household’s assets.

- **Figure** 5 summarizes the results of the statistical test.
  - A light gray point indicates that the 1998 surface is significantly above the 1993 surface;
  - a darker gray point indicates that the 1998 surface is significantly below the 1993 surface;
  - a black point indicates that they are statistically indistinguishable at the five-percent significance level.
- Conclusion: no robust poverty dominance result.

3. **Table** 2 gives the results for tests of the differences in the dominance surfaces for stunting and child survival probability in Cameroon and Madagascar.
4. **Table** 3 shows tests of the differences between first-order dominance surfaces for stunting and child survival probability, in Colombia and the Dominican Republic.

Bounds to multidimensional dominance

- *Critical* poverty frontiers: bound the area of poverty frontiers which may not be exceeded for a robust multidimensional ordering of poverty to be possible.
- Figure 6 shows two critical poverty frontiers, for the $\Pi^{1,1}$ and $\Pi^{2,2}$ classes, respectively) for Uganda rural Eastern residents urban Northern residents. Up to these critical frontiers, poverty is lower in rural Eastern Uganda

Figure 1: Union and intersection poverty indices

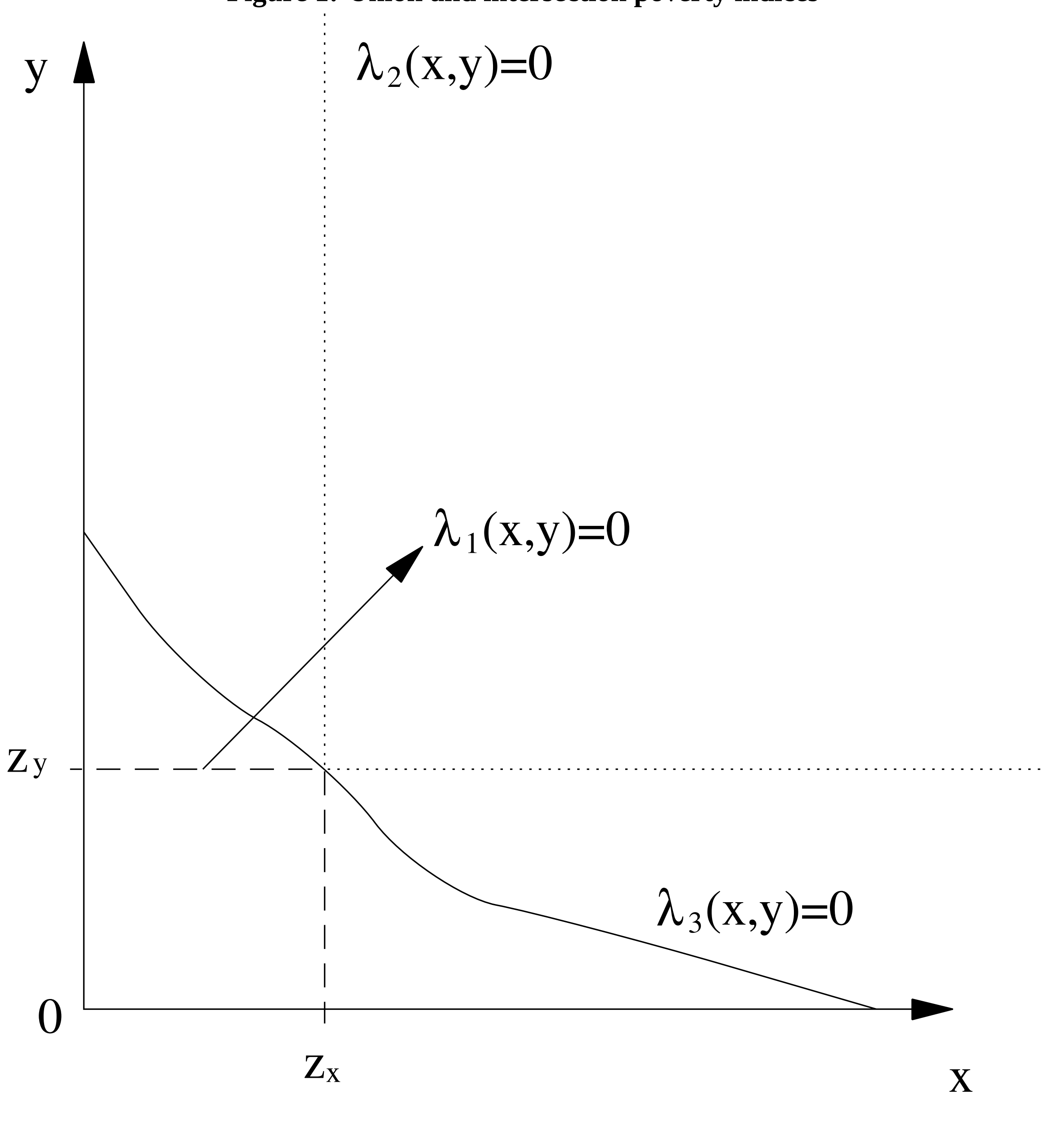

Figure 2: Dominance surface for Ghanaian children, 1989

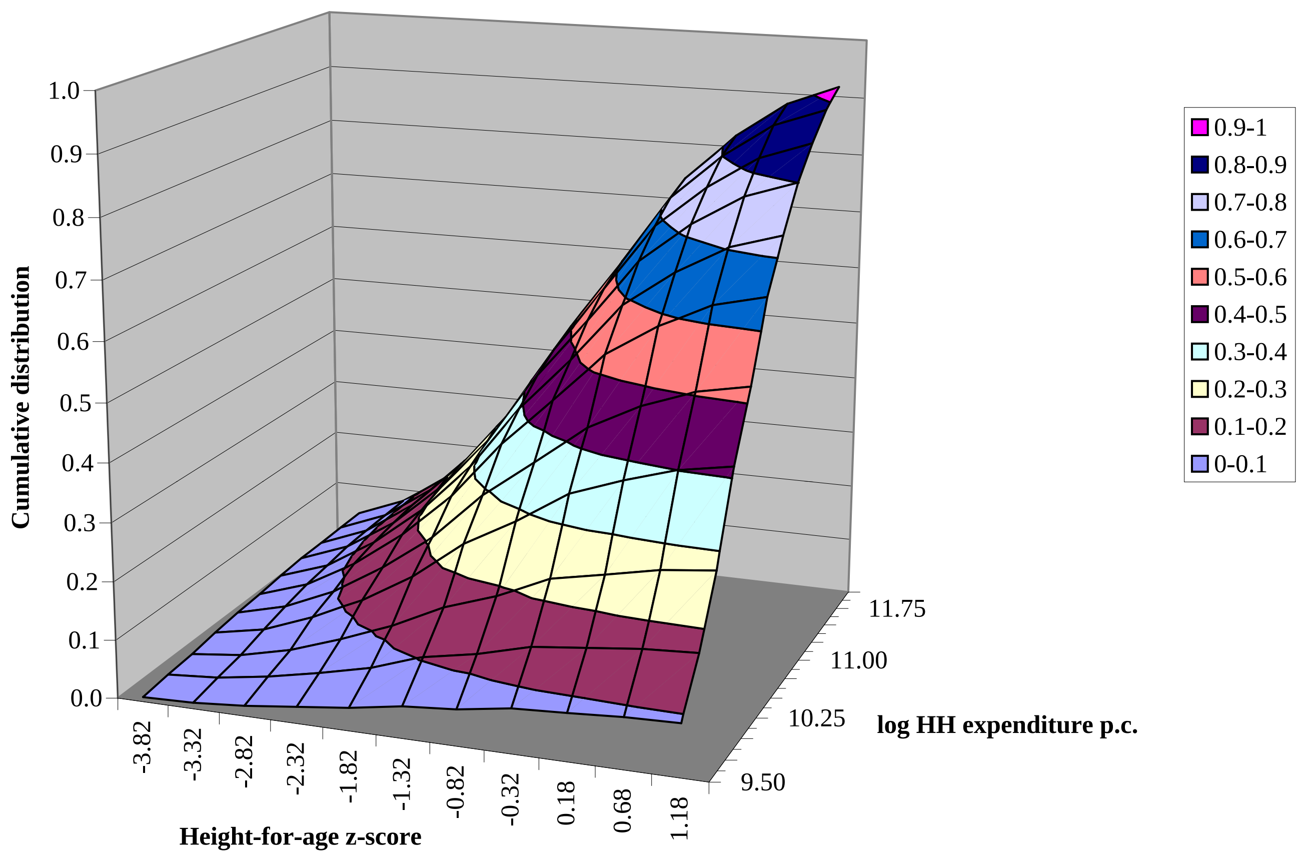

Figure 3: Example of difference in dominance surfaces, intersection dominance without marginal dominance

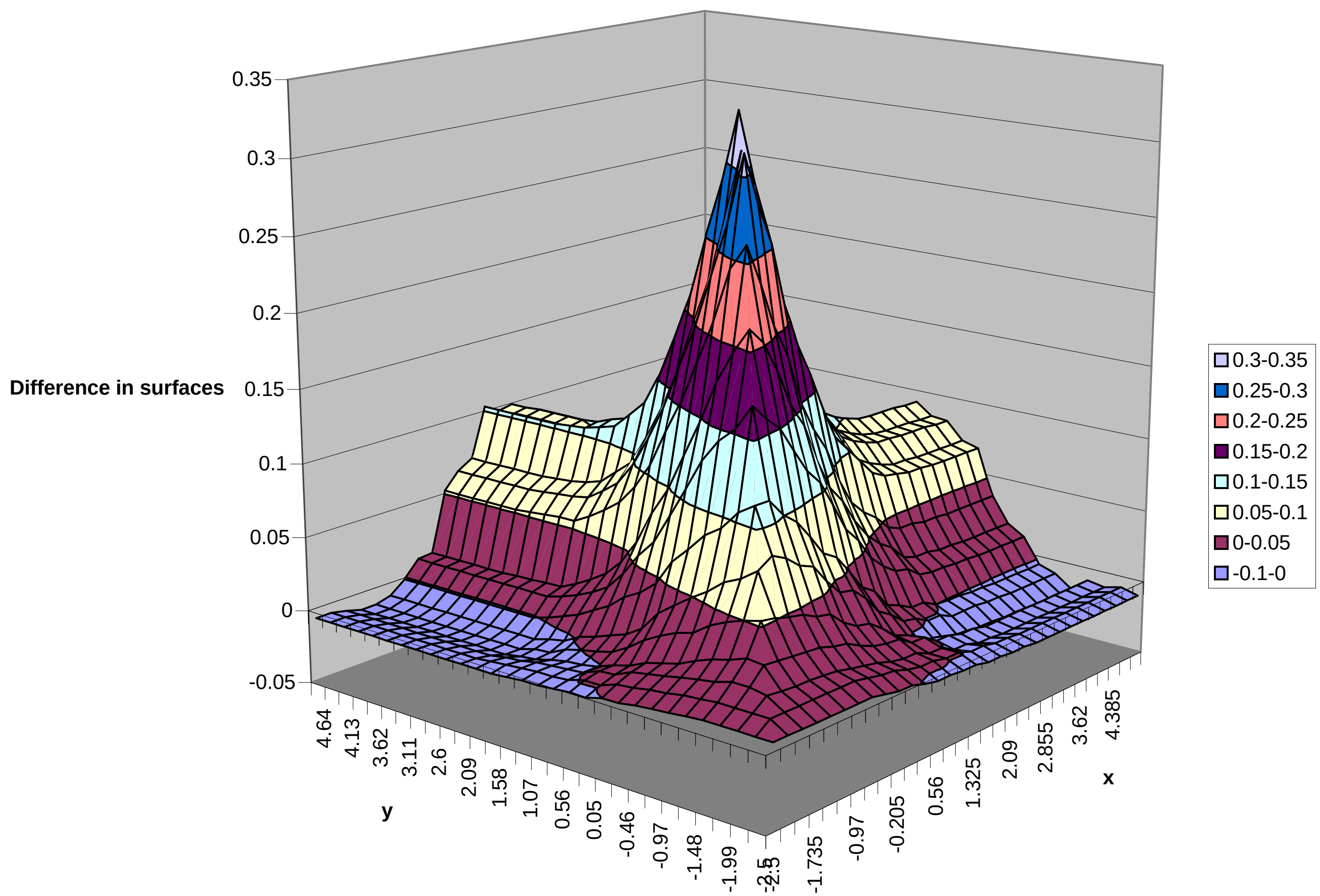

Figure 4: Urban minus Rural Dominance Surface for Viet Nam

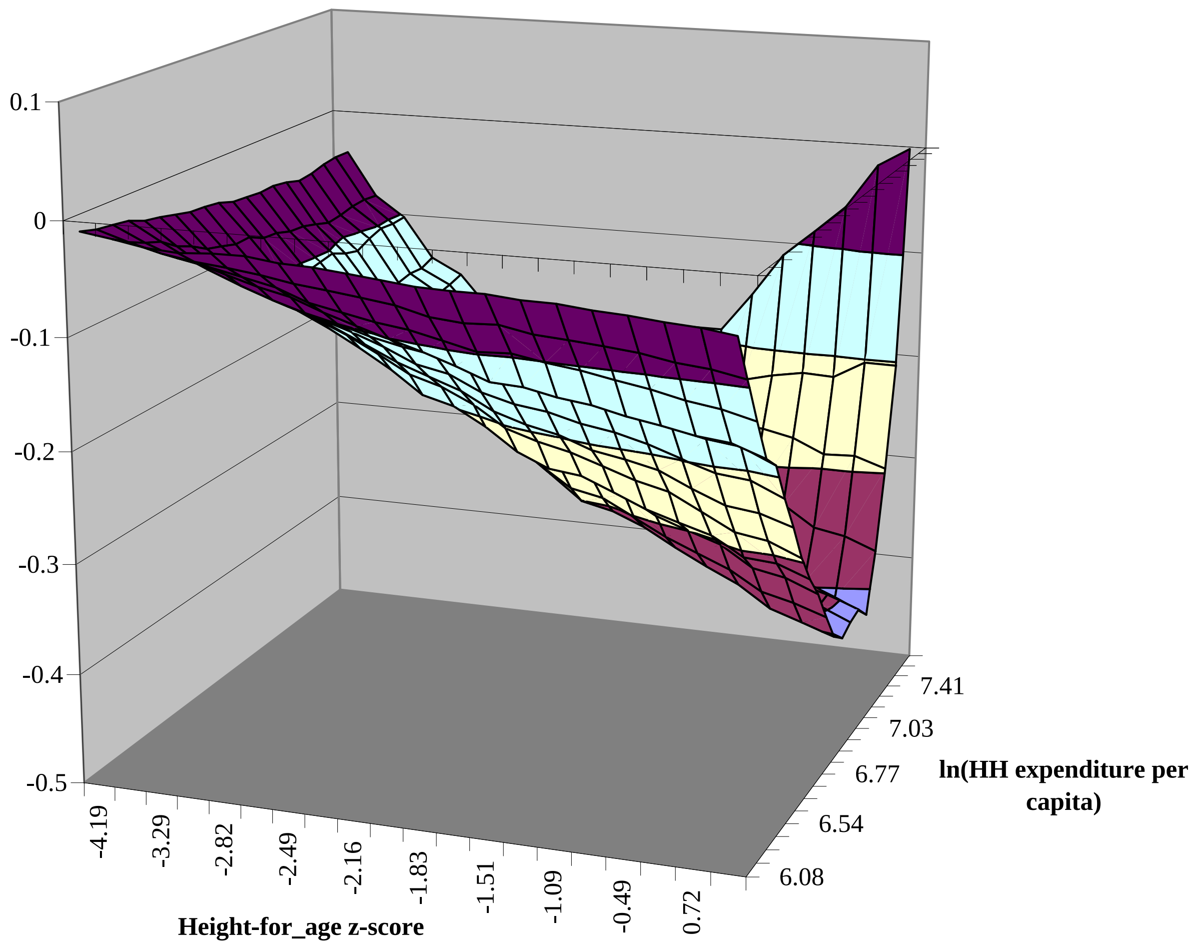

Figure 5: Test results for difference between 1993 and 1998 first-order dominance surfaces for Ghanaian children

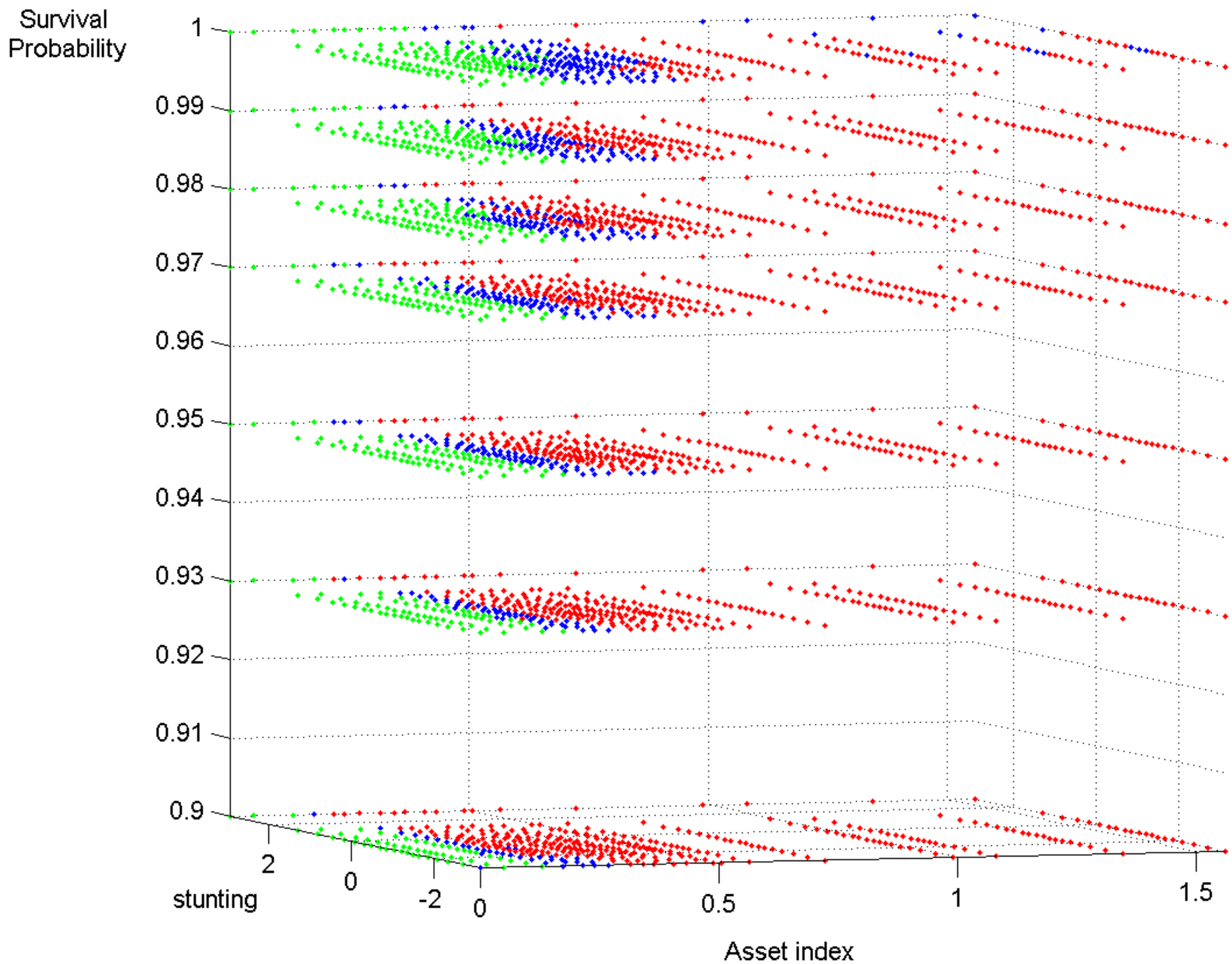

Figure 6: Critical Poverty Frontier, Rural Eastern region *vs.* Urban Northern region in Uganda (critical frontier minus two standard deviations)

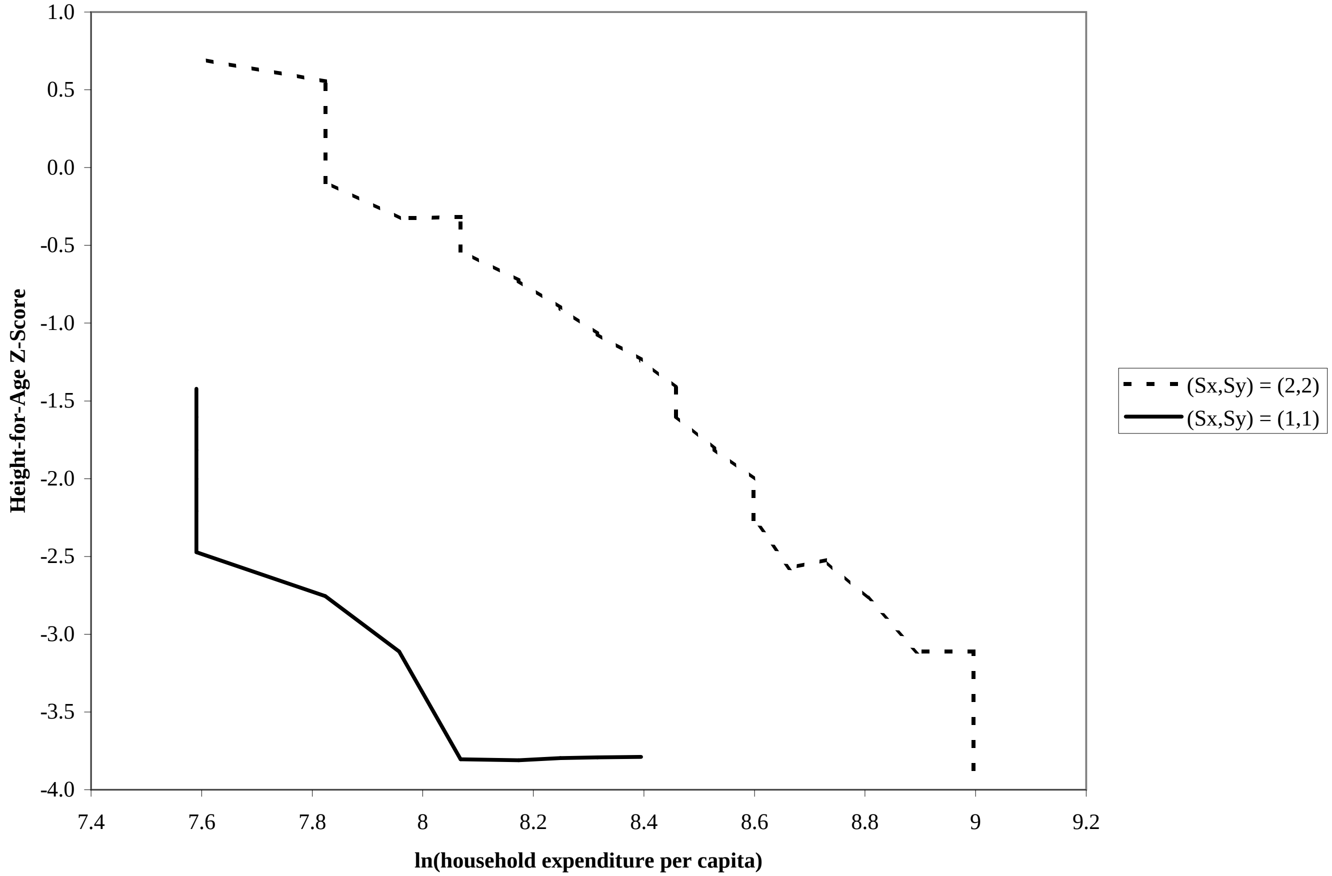

Figure 7: Domain for dominance testing

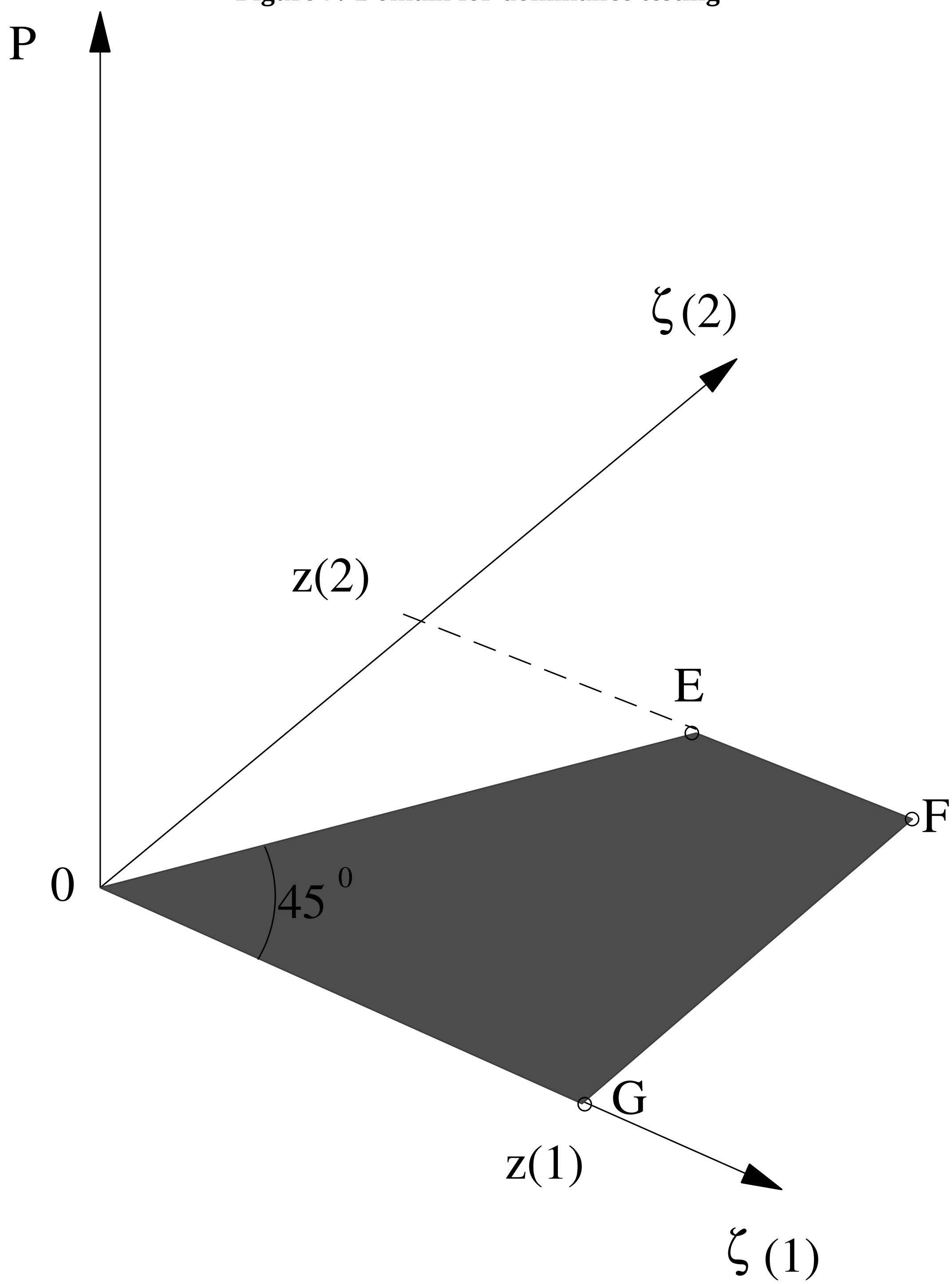

Table 1: Test results for difference between dominance surfaces for urban and rural children in Viet Nam, 1993

log of household expenditure per capita \ height-for-age z-score

	-4.19	-3.64	-3.29	-3.02	-2.82	-2.66	-2.49	-2.31	-2.16	-2.00	-1.83	-1.67	-1.51	-1.32	-1.09	-0.84	-0.49	0.01	0.72	5.47
6.08	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.26	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.37	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.47	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.54	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.61	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.66	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.71	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.77	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.84	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.89	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
6.95	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.03	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.08	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.27	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.41	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.59	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
7.88	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
9.41	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0

Notes: Sx=1, Sy=1

A negative sign indicates that the urban dominance surface is significantly below the rural one, a positive sign indicates the opposite, and a zero indicates that the difference is not statistically significant.

Table 2: Test results for difference between dominance surfaces for children in Cameroon and Madagascar, 1997

Height-for-age z-score \ Survival probability

	0.83	0.86	0.88	0.89	0.90	0.91		0.99	1.00
-4.19	-	-	0	0	-	-	…	-	-
-3.66	0	0	0	0	-	-	…	-	-
-3.35	0	-	-	-	-	-	…	-	-
-3.13	0	-	-	-	-	-	…	-	-
-2.88	0	-	-	-	-	-	…	-	-
-2.66	-	-	-	-	-	-	…	-	-
-2.50	-	-	-	-	-	-	…	-	-
	…	…	…	…	…	…	…	…	…
0.46	-	-	-	-	-	-	…	-	-
5.39	-	-	-	-	-	-	…	-	.

Notes: 1/ Sx=1, Sy=1
2/ A negative sign indicates that Madagascar's dominance surface is significantly above Cameroon's, a positive sign indicates the opposite, and a zero indicates that the difference is not statistically significant.
3/ The ellipses indicate that all intervening signs are negative.

Table 3: Test results for difference between dominance surfaces for children in Colombia and the Dominican Republic, 1995 and 1996

Height-for-age z-score \ Survival probability

	0.906	0.927	0.938	0.947	0.953		0.985	0.987	0.989	0.991	0.995	1.000
-2.85	-	-	-	-	-	…	-	-	-	0	0	0
-2.36	-	-	-	-	-	…	-	-	-	-	0	0
-2.07	-	-	-	-	-	…	-	-	-	-	0	0
-1.85	-	-	-	-	-	…	-	-	-	0	0	0
-1.67	-	-	-	-	-	…	-	-	-	0	0	0
-1.47	-	-	-	-	-	…	-	-	-	0	0	+
-1.33	-	-	-	-	-	…	-	-	-	0	0	+
-1.17	-	-	-	-	-	…	-	-	-	0	+	+
-1.04	-	-	-	-	-	…	-	-	0	0	+	+
-0.92	-	-	-	-	-	…	-	-	-	0	0	+
-0.76	-	-	-	-	-	…	-	-	-	0	+	+
-0.62	-	-	-	-	-	…	-	-	-	0	+	+
-0.49	-	-	-	-	-	…	-	-	-	0	+	+
-0.35	-	-	-	-	-	…	-	-	-	-	+	+
-0.12	-	-	-	-	-	…	-	-	-	-	+	+
0.07	-	-	-	-	-	…	-	-	-	-	0	+
0.34	-	-	-	-	-	…	-	-	-	-	0	+
0.68	-	-	-	-	-	…	-	-	-	-	0	+
1.05	-	-	-	-	-	…	-	-	-	-	-	+
5.92	-	-	-	-	-	…	-	-	-	-	-	0

Notes: Sx=1, Sy=1
A negative sign indicates that the Domincan Republic's dominance surface is significantly above Colombia's, a positive sign indicates the opposite, and a zero indicates that the difference is not statistically significant.
The ellipses indicate that all intervening signs are negative.

Table 4: t-statistics for difference between household income with child allowances *vs.* with social security (Romania)

Household income \ Household size	6 or more	5 or more	4 or more	3 or more	2 or more	1 or more
36,316	-30.51	-26.01	-20.24	-9.68	21.25	32.80
46,630	-36.27	-30.34	-24.34	-11.96	20.14	31.48
59,874	-41.95	-36.41	-29.30	-15.76	18.02	27.29
76,880	-47.80	-41.96	-34.84	-20.38	13.75	19.26
98,716	-54.91	-47.82	-39.52	-24.29	7.39	9.47
126,750	-57.50	-50.75	-42.30	-27.13	0.45	1.75
162,750	-59.59	-52.29	-45.60	-30.02	-10.08	-8.35
208,980	-47.90	-45.00	-42.05	-29.21	-15.98	-13.77
268,340	-38.35	-36.73	-35.02	-27.07	-17.62	-15.56
344,550	-27.02	-25.99	-25.41	-19.47	-13.52	-11.95
442,410	-17.74	-18.26	-17.04	-13.60	-8.63	-7.41
568,070	-18.13	-11.28	-10.25	-7.50	-4.46	-3.76
729,420	-7.23	-7.55	-7.58	-7.01	-2.68	-2.29
936,590	-4.30	-3.70	-3.26	-1.81	-0.25	-0.23
1,202,600	-10.34	-5.66	-3.48	-1.65	-0.07	-0.06
1,544,200	-7.86	-3.89	-2.17	-1.23	0.37	0.33

Notes: s=1. Results are similar for s=2 and s=3.
A negative sign indicates that income with child allowances dominates

Table 5: t-statistics for difference between per capita expenditures for literate and illiterate Peruvians, 1985 minus 1994

Household income \ Literacy	Illiterate	Literate
403	-1.95	-3.21
518	-4.93	-5.76
665	-7.69	-8.35
854	-14.93	-15.33
1,097	-22.37	-24.37
1,408	-28.97	-31.28
1,808	-35.47	-38.95
2,322	-41.48	-46.19
2,981	-46.16	-51.91
3,828	-48.38	-53.91
4,915	-49.63	-55.40
6,311	-46.49	-51.90
8,103	-40.41	-45.30
10,405	-35.02	-39.00
13,360	-26.61	-29.54
17,154	-21.45	-23.74
22,026	-16.02	-17.51

Notes: s=1.
A negative sign indicates that household expenditures in 1985 dominate those in 1994, and vice-versa.

Figure 8: A correlation-increasing switch