DEVELOPING A GLOBAL MODEL FOR TRADE, FINANCE AND INCOME DISTRIBUTION

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Francis Cripps
Alphametrics Ltd

Alex Izurieta
Cambridge Endowment for Research in Finance

and

Terry McKinley
International Poverty Centre
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Francis Cripps; Alex Izurieta** and Terry McKinley***

1 INTRODUCTION

This paper is the product of a research programme, called State of the World Economy (SOWE), which has been sponsored by the International Poverty Centre, Brasilia (IPC) and the Cambridge Endowment for Research in Finance (CERF). Alphametrics in Thailand has played the leading role in originating and developing the research programme.¹

The core of the programme is an integrated databank and modelling framework whose purpose is to identify sources of imbalances in the world economy and examine the potential impact of changing policies and trends.

The programme is motivated by the observation that the scale and significance of global developments is unlike anything experienced in the past. The uniqueness of the current situation results from a long period of liberalization of international trade and finance, which has resulted in unprecedented cross-border flows of goods, services, income and investment and, in the case of many of the largest countries, massive accumulation of external assets and liabilities. Although the outcome has been beneficial for a proportion of the world’s population, the impact has been adverse or much less favourable for others. Moreover, new risks have emerged that concern people in all countries, including those that have benefited the most.

Many institutions and research groups are nowadays focussing on various aspects of the global economy but there is no established framework that brings together analysis in different fields in order to clarify the potential impact of current developments and evolving government policies on global income distribution and human welfare in the medium to long term.

The SOWE programme seeks to provide a common framework that may be examined and refined from the perspective of different countries and regions of the world. Starting with income, population, trade and energy, the intention is to extend the analysis to include financial market linkages, the role of government budgets and the sectoral and functional distribution of income and employment.

The next three sections explain the motivation and theoretical basis of the model. Section 2 reviews examines briefly the recent evolution of globalization. Sections 3 and 4 review accounting identities and policy constraints that underlie the relationship between income, expenditure and accumulation of financial assets in the global system as a whole and within individual countries.

¹ Alphametrics Ltd.
² Cambridge Endowment for Research in Finance.
³ International Poverty Centre.
The final section specifies the quantitative macro-economic model that is being developed for the analysis of current global developments and the elaboration of medium-term prospects and policy scenarios. The model and its generation of policy scenarios are a work in progress. The initial version covers, trade, income, population and energy. A new version, soon to be released, will incorporate real exchange rates and a more comprehensive treatment of the balance of payments. This paper provides the initial blue-print for the further development of the model, which will include interest rates, stock market prices, monetary aggregates and wealth.

2 THE CONTEXT OF GLOBALIZATION

Throughout the 1990s the general mood about globalization was one of exhilaration. Although Japan was traversing one of the longest depressions known in post-war history and continental Europe was troubled by high rates of unemployment, international views about global economic performance were shaped by the boom in the US and its role as driver of global growth.

Hardly any influential mainstream economist questioned the rosy picture of more rapid economic growth combined with financial stability.2 U.S. public sector deficits were shrinking (and moving, in fact, into surplus at the end of the decade), wealth holders were getting richer, the GDP trend continued to be promising, workers’ salaries advanced faster than inflation and U.S. unemployment was at its lowest level in post-war history. Developing economies were also growing, with Asian economies in particular being regarded as a miracle of market-driven prosperity.3 In sum, the paradigm was working.

Confidence in the resilience of global markets was apparently unshaken in spite of a succession of financial crises (the U.K.’s ‘Black Wednesday’, 1992; Mexico, 1994; Southeast Asia, 1997; LCTM in the U.S., 1998; Russia, 1998; and Brazil, 1998). Perhaps the most critical setback in this period was the US recession in 2000-2001. But this setback was rapidly remedied with an unprecedented dose of fiscal expansion—a policy stance that occasioned, surprisingly, little or no objection.4

Nevertheless, global imbalances were growing throughout this period. The United States is now experiencing by far the largest external deficit ever seen (more than US $ 800 billion per year) and the deficits of some other high-income countries, such as Spain, Australia and the United Kingdom, are also unprecedented. The private sectors of many developed countries, including those in the US, Europe and more recently Japan, have moved from historic positions of net financial accumulation (surplus) to net borrowing (deficit), a transformation that is particularly striking for the household sector.

As a consequence, the ratio of household debt to income has reached exceedingly high levels; the net liability position (external debt) of several developed countries is now substantial. Deficits of high-income debtor countries are matched by surpluses of a handful of countries that are new industrial exporters in East Asia, are closely linked to the U.S. market (Canada, Japan, Latin America) or are energy exporters (Russia, countries in the Middle East or North Africa).

More than in preceding decades, international and development economists have expressed grave concern about the direction that the world economy has taken. The potential difficulties implied by the continuation of current global imbalances and the risks associated
with their correction are now widely recognized (IMF, 2006, 2007; UN-DESA, 2007; OECD, 2006; Eatwell, Cripps and Izurieta, 2005). Even if there is no agreement about the causes of the current imbalances, it is generally recognized that liberalization on a global scale might have outrun the capabilities of governments and international institutions to manage the consequences. While there is no consensus on possible solutions, the need for some degree of international co-ordination and policy intervention is increasingly acknowledged.

This new global landscape provides greater opportunity for meaningful dialogue aimed at promoting cooperation to identify feasible solutions. There is therefore a renewed interest in modelling frameworks that could be used to examine the complex interactions among different countries and world regions and assess the potential impact of major policy changes. That is why we believe that the research agenda of the State of the World Economy project could play an important role in informing both international and national policymaking. The focus of this project is building national policymaking capacities in developing countries to formulate intelligent national and regional policy responses to the trends in global imbalances and their potential impact, both positive and negative.

3 MARKET OUTCOMES AND NATIONAL POLICIES IN A GLOBAL SYSTEM

Any global macro-model that will be used to examine linkages between trade, finance and income must take account of accounting identities that link income, expenditure, accumulation of financial assets, the balance of payments and exchange rate movements within a global economy that is ultimately a closed system.

3.1 ACCOUNTING IDENTITIES

Consider the financial position of a single entity (bloc, country or sector).

Net acquisition of financial assets $NA$ is the difference between income $Y$ and expenditure on goods and services $H$:

$$[1] \quad NA_t \equiv Y_t - H_t$$

The financial position $A$ (stock of assets net of liabilities) at the end of each period is given by the starting position $A_{t-1}$ plus net acquisition and holding gains $GA$:

$$[2] \quad A_t \equiv A_{t-1} + NA_t + GA_t$$

Holding gains on financial assets can be expressed as the current rate of price change $\alpha$ multiplied by the inherited stock:

$$[3] \quad GA_t \equiv \alpha_t A_{t-1}$$

Thus:

$$[4] \quad A_t \equiv (1 + \alpha_t)A_{t-1} + NA_t$$
The value of tangible assets $K$ at the end of each period is given by the position at the end of the prior period $K_{t-1}$ plus net investment $I$ (purchases less sales and consumption of fixed capital) and holding gains $GK_t^*$:

$$K_t = (1 + \lambda_t)K_{t-1} + I_t$$

where

$$GK_t = \lambda_t K_{t-1}$$

and $\lambda_t$ is the rate of price change of the portfolio of tangible assets in each period.

Finally, wealth comprises tangible assets as well as the financial position:

$$W_t = K_t + A_t = (1 + \lambda_t)K_{t-1} + I_t + (1 + \alpha_t)A_{t-1} + NA_t$$

### 3.2 The Relation Between Expenditure, Income and Financial Wealth in a Closed System

$H$, spending on goods (including gross investment) and services in each country, is determined on the basis of financial objectives for accumulation of net assets $NA_t^*$ and assumptions about income $Y^*$:

$$H_t = Y_t^* - NA_t^*$$

The combined financial objective $NA_t^*$ may itself derive from sector targets for wealth $W$ or net financial assets $A$ with or without allowance for holding gains.

For the world as a whole, income $Y$ is equal to spending. Adding over all countries $j$:

$$Y_t = \sum_j H_{jt} = \sum_j Y_{jt}^* - \sum_j NA_{jt}^*$$

By implication, aggregate income of the world as a whole tends to exceed or fall short of assumptions depending on whether financial objectives (represented by the expected figure for net acquisition of financial assets) summed over all countries comes to a negative figure (deficit) or a positive one (surplus). In effect, the aggregate of country-level objectives imparts an upward or downward bias to movements of aggregate demand in the world as a whole.

The central proposition of this modelling framework should now be underlined. If behavioural relations determining ex-ante expenditure decisions and ultimately global income are properly taken into account, there is no guarantee that the final outcome would be an ‘equilibrium’ condition in any meaningful sense. The combined result of assumptions and expectations about disposable income, financial savings and market valuations would not necessarily (and would usually not) lead to full employment of resources and price stability. Thus, there is scope for government policy. Moreover, there is a limit to which the policies of blocs and countries taken separately can achieve significantly better outcomes unless there is some form of international co-ordination.
Of course, income assumptions $Y^*$ are influenced by a range of factors, including the strength of a country's trade balance and productive potential as well as its financial position. Therefore, even if some countries are ready to incur net deficits, pushing income above other countries' assumptions, it does not follow that growth of world income will accelerate significantly.

There are two different ways in which the country-level financial objective (presumably the sum of targets for $NA^*$) and the income assumption $Y^*$ can be interpreted.

On the one hand, aggregates at country level may be regarded as the sum of components determined separately by the main actors - households, firms, government and central bank. Even if there is some convergence of assumptions about national income $Y^*$, the financial objectives of agents in each sector may go in different directions; the combined outcome $NA^*$ is nothing more or less than the sum of individual objectives. For example, households may be predominantly concerned with accumulating financial wealth (thus saving), firms with augmenting fixed capital (thus investment), government with pursuing social stability by means of taxation and public spending and the central bank with accumulating reserves. To model the joint outcome, it is necessary to model the behaviour of each sector.

On the other hand, it is often the case that government and the central bank (the 'authorities') take responsibility for the national outcome through active fiscal and monetary policies targeted at objectives such as growth of employment and income and stability of prices and exchange rates. In this case, the combined financial objective $NA^*$ at the level of the national economy may be determined, explicitly or implicitly, by fiscal and monetary policies that adjust or compensate for the autonomous behaviour of households, firms and government. However, such compensation would unfold at the national or regional level, with partial influence, at best, on the global outcome, and with hardly any scope to capture feedback effects on the level of domestic activity and employment. Desired goals could be achieved more effectively by policy co-ordination mediated by the responses of international institutions.

3.3 POLICY REGIMES

This section considers alternative policy regimes, with particular emphasis on implications for the balance of payments.

3.3.1 Open system with price stability objective

The most common or at least most widely advocated regime is an open payments system with monetary policy targeted at price stability. Within this regime, the effectiveness of fiscal policy is subdued ex-ante.

In an open economy, internal price stability is at least partially dependent on the stability of the exchange rate, which in turn requires a sound external financial position. Either non-residents must be persuaded to invest in domestic assets or residents, including the government and central bank, must hold sufficient external assets to absorb fluctuations in external trade and income.

Theoretically, exchange rate devaluation could be acceptable if it does not result in domestic price inflation but in practice this rarely happens unless there is a simultaneous internal deflation of demand. Therefore, in practice monetary authorities are likely to curb
domestic credit creation if the real exchange rate moves outside a target range, whether because of domestic price inflation or nominal devaluation.

By implication, net acquisition \( NA^* \) and, equivalently, the current account \( B^* \) are maintained within limits that are judged to be conducive to monetary stability and an acceptable real exchange rate. Thus, government borrowing and net lending or borrowing by the private sector must be kept within limits that are consistent with objectives for net acquisition and the current account of the country as a whole. Usually, the fiscal stance is excessively tight, all the more when monetary and fiscal policies are determined independently from each other.\(^{12}\)

### 3.3.2 Open system with growth objective

Governments are concerned with demand management objectives as well as price stability and this concern may be reflected in the formulation of fiscal and monetary policy. If policy were targeted on demand management alone, net acquisition of financial assets by the government and the private sector combined would be calibrated to match the expected outcome for the current account \( BP \) when the economy is operating at full capacity utilization \( Y_F \).

In a liberalised open system, monetary policy targeted at a growth objective is viable so long as a number of financial conditions are satisfied:

- the current account and net external position of the country are conducive to exchange rate stability
- the real exchange rate does not become too much overvalued or undervalued
- the rate of price inflation remains low.

If the external position is too strong, there is a risk that the real exchange rate will be overvalued. In this case, the authorities must encourage resident investment abroad and discourage non-resident investment in the country while continuing to operate monetary policy in accordance with growth and stability objectives.\(^{13}\)

If the real exchange rate becomes overvalued because of internal price inflation, the authorities must seek to bring about exchange-rate devaluation without triggering an inflation-devaluation spiral. If management of the exchange market is successful in this sense, the full-employment current account \( BP \) may strengthen, implying some tightening of monetary policy.

If the external position weakens progressively (leading to a deteriorating \( BP \) and eventually a growing debt position), confidence of external investors may eventually dissipate since the exchange rate, nominal and real, no longer appears sustainable. In this case, the growth objective must be abandoned and monetary policy will switch to the stability objective, with an initial deflationary correction intended to restore confidence in the exchange rate and prevent any inflationary spiral.

### 3.3.3 Managed systems

When a country's external position is very weak, it becomes difficult to introduce or maintain an open system.
If the exchange rate is largely influenced by international sentiments, attempts by the
government to influence exchange rate movements may result in perverse outcomes, such as
an inflation spiral and external debt overhang or stabilization accompanied by internal
recession. When the external constraint is binding, export growth may itself be constrained
by lack of imported supplies while foreign investors, creditors and donors are unwilling to
advance funds. In this context, freedom of capital movement may simply result in speculation
and capital flight. It may not be possible for domestic deflation to restore external balance
and the country may be in danger of reaching a point at which scope for recovery is fatally
compromised by depletion of productive potential, postponed investment and break-up of
the social fabric.

In the face of persistent shortages of foreign exchange, a government must restrict its use
to essential debt service and purchases of imports. The foreign exchange position is managed
directly by the central bank and the domestic financial system is effectively isolated from
international financial markets. Full employment remains a remote goal but a liberalized open
regime is likely to make matters worse.14

This is still the position of many low-income countries in the world today.

3.3.4 Trade-offs in the absence of international co-ordination

When a country has a strong trading position it could achieve full employment with an
external surplus, implying its accumulation of external assets, public or private. In this case,
the growth objective prevails.

Another case in which the growth objective might prevail is that of a country (such as
the USA) that has very strong external borrowing power. In this case, the authorities may
courage domestic borrowing in order to sustain high employment in the face of continuing
external deficits.15 They may expect that continued deficits will eventually result in
depreciation of the exchange rate, leading to an improvement in the trade balance and
stabilization of external debt. However, the most likely breaking points are

- inability or reluctance to engender private and public debt on the scale necessary
to maintain full employment (implying recession)
- refusal of external investors to acquire domestic assets at the current exchange
rate (implying devaluation).

There may be a soft landing somewhere between these two boundary regions, in which
real devaluation would be accompanied by modest recession and minimal inflation. A soft
landing would present the authorities in the USA with trade-offs similar to those faced by
policy-makers in other high- and middle-income countries.

If the trading position of a country is not so strong and its borrowing power not certain,
the authorities have to manage aggregate demand at an intermediate position between a
level that would achieve full employment and a level consistent with a strong external
financial position. This is the position of many developed and developing countries.

The trade-off chosen by each country is important for the world as a whole since, as
mentioned in the preceding section, the sum of financial targets for the world as a whole
exerts an upward or downward momentum on aggregate demand.
Even more critical for many developing regions than the pace of global trade and income is the contradictory character of market forces, which makes it difficult for countries with weak external positions to sustain a development process. A limited group of rich countries has mature and liquid capital markets, which attract the bulk of financial investment. Other countries that find success with export-orientated growth strategies must maintain ongoing production cost advantages or risk losing their position to cheaper competitors.

For the large number of countries that do not fit into either of the categories above, the capital that would be required for them to invest in sustained development based on integration into the world economy is not available. This argument is fully elaborated in McKinley (2006), which calls for a radical turn-round of the current policy paradigm, with the implication that rich countries would have to restructure their expenditures instead of seeking to impose conditions on poorer economies and blaming their own imbalances on middle-income countries that are implementing successful strategies of development.

4 THE COMPOSITION OF ASSETS AND LIABILITIES

4.1 BALANCE SHEETS OF HOUSEHOLDS AND BUSINESSES

There are major differences between countries in the amount and composition of private sector assets and liabilities, reflecting differences in institutions and the level of economic development.

The most important differences for households concern

- mortgage borrowing linked to the value of real estate
- diversification of financial wealth accumulated in pension and insurance funds.

The main differences affecting firms are

- the role of stock markets and bond markets in corporate finance
- the level of involvement of multinational corporations.

Changes in the above can have a significant impact on net acquisition of financial assets by the private sector as a whole.

4.2 DOMESTIC IMPACT OF HOLDING GAINS

Changes in asset prices are relevant to the determination of demand and income to the extent that they influence the net acquisition of financial assets. The volume of holdings is an increasingly relevant factor too since the process of financial liberalization has led to an unprecedented, widespread accumulation of financial wealth.16

The most important domestic consequence of asset price movements is the impact of real estate prices on mortgage borrowing by households and firms. It can readily be demonstrated that the real estate market has had a major impact on demand in many countries during various historical periods.17
So long as holding gains remain favourable to net debtors, there is little incentive for policy-makers to intervene. For example, in the US, the personal sector has been the driver of growth by increasing its debt position from about 1.1 times its annual income in the early 1990s to more than 1.6 times at present. Both the former and current chairs of the Federal Reserve have declared that there is no cause for concern since the net worth of the personal sector has been relatively unaltered (in virtue of the rise of asset prices).\(^\text{18}\)

Another potential influence on domestic spending is fluctuations in stock market prices. These tend to be pro-cyclical; moreover, and it is not easy to separate cause from effect. Although the scale of notional gains and losses is huge, fluctuations in the value of equity holdings appear to be reflected in wealth rather than short-term adjustments of expenditure or other financial positions. In addition, in the long run stock market value is linked to the value of underlying assets and liabilities of firms. Therefore, any decisive impact of stock market volatility on expenditure is questionable.\(^\text{19}\)

### 4.3 IMPACT OF HOLDING GAINS ON THE EXTERNAL POSITION

A country’s external position is exposed to movements in financial markets as follows:

- the real value of the net foreign currency position is affected by exchange rate movements
- the value of direct investment abroad and resident holdings of foreign equity is additionally affected by foreign business conditions and movements in foreign stock markets
- the value of non-resident direct investment and holdings of domestic equity is affected by domestic business conditions and movements in the domestic stock market.

It is also necessary to point out that markets for global derivatives provide ever-increasing opportunities for investors to hedge against instrument-specific and currency-specific risk. This should help reduce the impact of cross-border holding gains.

At a country level, the combined effect of price changes in financial markets over a period of one or two years can be equal to several percentage points of GDP, comparable in magnitude to the current account surplus or deficit.\(^\text{20}\) To the extent that it is sensible and feasible to value a country’s external assets and liabilities at current market prices, price and exchange rate fluctuations do inject substantial volatility into year-on-year movements of the net external position. Holding gains, such as those experienced by the USA, might cloud the policy perspective and encourage decision-makers to take a relaxed view of the risks implied by continuing deficits.\(^\text{21}\)

However, in the longer run internal and external stock market indexes tend to move together and the exchange rate tends to offset changes in domestic and international price levels. If price movements are consistent in this sense, the long-term impact on the net external position is less important. For holding gains to produce a continuing effect, divergent changes in asset prices would have to continue over a sustained period of time. In the case of a debtor country, price movements that continue to favour the debtor will take their toll on creditors and must ultimately discourage them from investing additional funds.
Needless to say, holding gains are not true cash-flow income. They are accounting changes in the balance sheet that result from imputing end-of-period market values to the existing stock of assets. They induce a perception of wealth based on the assumption that if the holdings were sold today at the current price, they would yield the estimated gain or loss. In order to realize holding gains or losses, a ‘sell-buy’ transaction has to take place. Thus, it is not unreasonable for government authorities and investors to discount short-term fluctuations in market value to some extent when assessing a country’s financial position.

5 OUTLINE OF A FORMAL MACRO MODEL

In the following, variables denoted by capital letters represent time series. Estimated intercepts, usually bloc-specific, are denoted $b$ or $b_j$. The symbol $u$ denotes an equation residual modelled by a time series process (generally AR(1)). Imposed coefficients in each equation are denoted $c_0$, $c_1$ etc. Subscript $j$ denotes a bloc, $k$ a partner bloc and $t$ a time period.

Money variables are measured in million US dollars, deflated by a domestic or international price index to adjust for inflation. Domestic income, expenditure and asset values are divided by the domestic expenditure deflator scaled to allow for base-year differences in purchasing power parity (PPP).

Balance of payments flows and stocks of external assets and liabilities measured in current dollars are deflated by the dollar price index for world exports of manufactures to obtain series denominated in what will be termed international purchasing power (IPP).

The real exchange rate converts between values measured in terms of domestic purchasing power (PPP) and values measured in terms of international purchasing power (IPP).

Asset prices and commodity prices are deflated by the relevant domestic or international deflator to derive ‘real’ prices. In the following, index numbers and rates are expressed in natural units (ratios) rather than percentages to avoid the need to divide by 100.

5.1 THE BALANCE OF PAYMENTS

The population of each bloc and its real exchange rate are treated as exogenous variables.

\[
N_{jt} \quad \text{population \ (millions, exogenous)} \\
RX_{jt} \quad \text{real exchange rate \ (index, exogenous)} \\
RXLS_{jt} \quad \text{lagged real exchange rate \ (log)}
\]

\[
RXLS_{jt} = c_0 \log(RX_{jt}) + c_1 RXLS_{j,t-1} \quad \text{where } c_0 = 0.3 \text{ and } c_1 = 0.7
\]

5.1.1 Primary commodities

Production and domestic absorption depend on domestic income, world prices, the exchange rate and other supply-side and demand-side factors not modelled explicitly.

WORLD VARIABLES

Global prices of food and raw materials and of energy result from market-clearing. The equations that follow describe smoothed price variables and demand and supply in each bloc.
\( PA_t \)  
real price of food and raw materials (index)  

\( PE_t \)  
real price of energy (index)  

\( PALS_t \)  
lagged price of food and raw materials (log)  

\( PELS_t \)  
lagged price of energy (log)  

\[
PALS_t = c_0 \log(PA_t) + c_1 PALS_{t-1} \\
PELS_t = c_0 \log(PE_t) + c_1 PELS_{t-1}
\]

where \( c_0 = 0.3 \) and \( c_1 = 0.7 \)

**BLOC VARIABLES**

An Engel’s curve is used to estimate domestic demand for food and raw materials. Production is equal to domestic demand plus the trade surplus on food and raw materials, adjusted for the terms of trade \( PA \). A protected component of domestic production \( AH \) is estimated as a proportion of production and domestic demand. It is assumed that protected production responds to changes in domestic demand, irrespective of world prices. Other components of production respond to world prices and are not affected by domestic demand.

\( AH_{jt} \)  
protected component of domestic production (IPP)  

\( AD_{jt} \)  
demand for food and raw materials (IPP)  

\( AP_{jt} \)  
production of food and raw materials (IPP)  

\( BA_{jt} \)  
trade balance for food and raw materials (IPP)  

\[
AH_{jt} = c_0 \frac{AD_{jt}}{AD_{jt} + AP_{jt}} AP_{jt} \\
\]

where \( c_0 = 0.7 \)

\[
AD_{jt} = c_0 N_{jt} + c_1 Y_{jt} \cdot \exp\left(-c_3 \frac{PALS_t}{RXLS_{jt}}\right)
\]

where \( c_0 = 150, c_1 = 0.05 \) and \( c_3 = 0.3 \)

\[
\Delta AP_{jt} = AH_{j,t-1} \frac{\Delta AD_{jt}}{AD_{j,t-1}} + \left(AP_{j,t-1} - AH_{j,t-1}\right) \left(b_j + c_0 \frac{PALS_t}{RXLS_{jt}} + u_{jt}\right)
\]

where \( b_j \) is an estimated intercept and \( c_0 = 0.8 \)

\[
BA_{jt} = PA_t \left(AP_{jt} - AD_{jt}\right)
\]

Energy production and use are measured in million tons of oil equivalent (mtoe). The energy model is similar to that for food and raw materials but with a unit income elasticity. In this case, physical data on production, exports and imports are available and domestic use is inferred from production less net exports. The relationship between net exports in physical
units and the dollar trade balance on energy products depends on the oil price and mix of products imported and exported by each bloc.

\[ EH_{jt} = \frac{ED_{jt} c_0}{ED_{jt} + EP_{jt}} \]  \hspace{1cm} \text{where } c_0 = 0.5

\[ \ln(ED_{jt}) = b_j + c_0 \ln Y_{jt} - c_1 \frac{PE_{jt}}{RXLS_{jt}} + u_{jt} \]  \hspace{1cm} \text{where } c_0 = 1; c_1 = 0.3

\[ \Delta EP_{jt} = EH_{j,t-1} \frac{\Delta ED_{jt}}{ED_{j,t-1}} + \left(EP_{j,t-1} - EH_{j,t-1}\right) \left( b_j + c_0 \Delta \frac{PE_{jt}}{RXLS_{jt}} + u_{jt} \right) \]  \hspace{1cm} \text{where } c_0 = 0.5

\[ BE_{jt} = b_j + c_0 PE_{jt} (EP_{jt} - ED_{jt}) \]  \hspace{1cm} \text{where } c_0 \text{ is the base-year price of oil in dollars per ton.}

WORLD MARKET

World prices are determined by solving for \( PA_t \) and \( PE_t \) such that

\[ \sum_j AP_{jt} = \sum_j AD_{jt} \]  \hspace{1cm} \[\text{[20]}\]

\[ \sum_j EP_{jt} = \sum_j ED_{jt} \]  \hspace{1cm} \[\text{[21]}\]

5.1.2 Manufactures and services

Imports depend on domestic income. Exports are demand-determined. Market shares are price-sensitive, responding to changes in real exchange rates.

BLOC VARIABLES

\( XM_{jt} \) exports of manufactures (IPP)

\( MM_{jt} \) imports of manufactures (IPP)
\( BM_{jt} \) trade balance on manufactures (IPP)  
\( SM_{jkt} \) share of manufactured exports of bloc \( j \) in the import market of bloc \( k \)  
\[
SM_{jkt} = \frac{\exp(b_{jk} - c_0 RXLS_{jkt} + u_{jkt})}{\sum_i \exp(b_{ik} - c_0 RXLS_{ikt} + u_{ikt})} \quad \text{where } c_0 = 3 
\]  
\[
XM_{j} = \sum_k SM_{jkt} \cdot MM_{kt} 
\]  
\[
\Delta \ln(MM_{j}) = b_{j} + \Delta \ln(H_{jt} + c_0 XM_{j}) + u_{jt} \quad \text{where } c_0 = 2 
\]  
\[
BM_{jt} \equiv XM_{jt} - MM_{jt} 
\]  
\( XS_{jt} \) exports of services (IPP)  
\( MS_{jt} \) imports of services (IPP)  
\( BS_{jt} \) trade balance in services (IPP)  
\[
XS_{jt} = \frac{\exp(b_{j} - c_0 RXLS_{j} + u_{jt})}{\sum_i \exp(b_{i} - c_0 RXLS_{it} + u_{it})} \sum_i MS_{it} \quad \text{where } c_0 = 1 
\]  
\[
\Delta \ln(MS_{jt}) = b_{j} + \Delta \ln(H_{jt} + MM_{jt} + XM_{jt}) + u_{jt} 
\]  
\[
BS_{jt} \equiv XS_{jt} - MS_{jt} 
\]  

### 5.1.3 Income flows, the current account and net external assets

The main components of income flows on the balance of payments are workers’ remittances, profits, dividends and interest. Workers’ remittances could be treated as an extension of trade in services while profits, dividends and interest should be modelled as returns on financial assets and liabilities. We will use a minimum specification that defines a feedback between accumulation of net external assets and liabilities and income flows proxied by the interest rate.

In the following, the net external asset position \( A \) is measured in international purchasing power (IPP).

**World variables**

\( Rl_t \) real interest (annual rate)  
\( RIL_t \) lagged real interest (annual rate)  
\[
RIL_t = c_0 Rl_t + c_1 RIL_{t-1} \quad \text{where } c_0 = 0.3; c_1 = 0.7
\]
**BLOC VARIABLES**

\( Bl_{jt} \) net income and current transfers (IPP)

\( Bj_{jt} \) current account balance of payments (IPP)

\( Aj_{jt} \) net external assets (IPP)

\[ BI_t = b_j + RIL_{jt} \cdot RX_{jt} \cdot A_{j,t-1} + u_{jt} \]  

\[ B_{jt} \equiv BA_{jt} + BE_{jt} + BM_{jt} + BS_{jt} + BI_{jt} \equiv NA_{jt} \]

Note that in a fully specified model, equation 2 can be written as

\[ 2a \quad A_{jt} \equiv A_{j,t-1} + B_{jt} + GA_{jt} \]

The equation determining holding gains on external assets and liabilities \( GA \) is specified further below.

**5.1.4 Direct investment**

Direct investment may be linked to trade or internal markets. The balance of payments impact is usually small because a large part of direct investment is financed by retained earnings or borrowing in the country where the investment is made. However, it is not possible to obtain a reliable net figure since *International Financial Statistics* does not distinguish related transactions in this way.

Direct investment can generate significant exposure to exchange rate movements and business conditions (proxied by stock market indexes), especially if the investment is not financed by borrowing in the same currency.

**5.1.5 Equity investment**

Like direct investment, equity investment by non-residents is a matter of mutual attraction. Equity portfolios are less stable. In small countries, inflows and outflows of external investment can have a substantial impact on the stock market. Considering the world as a whole, cross-border equity investment is a growing phenomenon but it is difficult to identify any specific influence of exchange rate movements on cross-border portfolios. Stock market prices are otherwise influenced by profits and therefore capacity utilization.

**WORLD VARIABLE**

\( PQ_t \) world equity price (dollar index)

\[ PQ_t = \sum_j w_j PD_{jt} RX_{jt} \]

where \( w_j \) is the weight of bloc j and \( PD_{jt} \) is an index of equity prices in bloc j.
BLOC VARIABLES

\( QF_{jt} \)  

direct investment abroad & resident holdings of foreign equity (IPP)

\( QH_{jt} \)  

non-resident direct investment & holdings of domestic equity (IPP)

\[ \ln QF_{jt} = b_j + c_0 \ln(RX_{jt}Y_{jt}) + c_1 \ln PQ_t + u_{jt} \]  

where \( c_0 = 1; c_1 = 1 \)

\[ \ln QH_{jt} = b_j + c_0 \ln(RX_{jt}Y_{jt}) + u_{jt} \]  

where \( c_0 = 1 \)

5.1.6 Other foreign currency investment and deposits

In an open system, foreign currency securities and deposits may be acquired by the private sector (e.g., through pension and insurance funds) as well as the monetary authority (through reserves). From the point of view of the country where funds are invested, liquidity is important since this helps to provide short-run stability of the exchange rate. So long as world markets provide a sufficient level of speculation, the exchange rate is determined by long-term expectations rather than current flows of trade, income and direct investment. Given domestic demand for foreign currency investments and deposits, the balancing item in the exchange account will be non-resident investments and deposits denominated in domestic currency.

BLOC VARIABLES

\( FF_{jt} \)  

other foreign currency assets of bloc j (net, IPP)

\( FH_{jt} \)  

other non-resident holdings of domestic currency assets (net, IPP)

\[ FF_{jt} = FF_{jt-1} + b_j + c_0 \Delta(RX_{jt}Y_{jt}) + u_{jt} \]  

where \( c_0 = 0.02 \)

\[ FH_{jt} = FF_{jt} + (QF_{jt} - QH_{jt}) - A_{jt} \]

5.1.7 Holding gains on external positions

WORLD VARIABLE

\( PXM_t \)  

deflator for world exports of manufactures (exogenous, dollar index)

BLOC VARIABLES

\( PH_{jt} \)  

domestic expenditure deflator (exogenous, domestic currency index)

\( GA_{jt} \)  

holding gain or loss on external position (net, IPP)
\[ GA_{jt} \equiv \left( \frac{PXM_{t-1}}{PXM_t} - 1 \right) FF_{j,t-1} - \left( \frac{RX_{jt} \cdot PH_{j,t-1}}{RX_{j,t-1} \cdot PH_t} - 1 \right) FH_{j,t-1} \]
\[
+ \left( \frac{PQ_{jt} \cdot PXM_{t-1}}{PQ_{j,t-1} \cdot PXM_t} - 1 \right) QF_{j,t-1} - \left( \frac{PD_{jt} \cdot PXM_{t-1}}{PD_{j,t-1} \cdot PXM_t} - 1 \right) QH_{j,t-1} \]

5.1.8 Monetary policy and domestic expenditure and income

**BLOC VARIABLES**

- \( Y_{jt} \) income (PPP)
- \( YP_{jt} \) productive potential (PPP)
- \( U_{jt} \) capacity utilization (ratio)
- \( H_{jt} \) domestic expenditure (PPP)

Assume that the authorities in each bloc manage fiscal and monetary policy in order to achieve some combination of growth and stability objectives. The authorities have to make assumptions about productive potential \( YP \) and the balance of payments and adjust interest rates and the government budget in order to safeguard the value of domestic currency. We represent the intended outcome of such policies by an expected level of capacity utilization \( U^* \) and domestic expenditure \( H^* \).

Given the policy stance represented by these variables, the outcome can depart from expectations on account of unanticipated changes in domestic expenditure \( H \) or the balance of payments \( B \). The balance of payments outcome has been modelled in some detail above. We assume that unanticipated changes in domestic expenditure are related to the level of income:

\[ H_{jt} = H_{jt}^* + c_0 \left( Y_{jt} - U_{jt}^* \cdot YP_{jt} \right) + u_{jt} \] where \( c_0 = 0.6 \)

The outcome for income and capacity utilization is determined by identities:

\[ Y_{jt} \equiv H_{jt} + B_{jt} / RX_{jt} \]

\[ U_{jt} \equiv Y_{jt} / YP_{jt} \]

Expected levels of capacity utilization \( U^* \) and domestic expenditure \( H^* \) may be formulated as the outcome of a trade-off between a balance of payments objective and a capacity utilization objective, responding to the need for monetary stability on the one hand and employment and income growth on the other. To simulate the trade off, we have to make assumptions about growth of productive potential and the balance of payments outcome consistent with different levels of capacity utilization.
The expected level of productive potential may be modelled using an adaptive formula responding to an assessment of prior-year capacity utilization and the trend growth rate:

\[ YP_{jt} = \left( \frac{Y_{jt-1}}{U_{jt-1}^*} \right) (1 + g_{jt-1}) \]

\[ U_{jt}^* = c_0 c_1 + (1 - c_0) U_{jt} \quad \text{where } c_0 = 0.5 \text{ and } c_1 = 0.95 \]

\[ g_{jt} = c_0 \left( \frac{Y_{jt}}{Y_{jt-1}} - 1 \right) + (1 - c_0) g_{jt-1} \]

We assume balance of payments expectations are based on the prior year outturn and an estimate of the response to changes in the level of capacity utilization. If the external position is weak, the authorities will adjust policy to achieve a certain degree of progress towards a satisfactory medium-term ratio of reserves or net external assets to income. If the external position is strong, policy-makers may pay more attention to the level of capacity utilization. The following reaction function is used to describe the trade-off between the two objectives (see the Appendix for details):

\[ U_{jt}^* = \frac{1}{2} (1 + q - r) \]

\[ q = U_{jt-1} + \frac{1}{m_{jt}} \frac{B_{jt-1}}{YP_{jt}} - \frac{1}{m_{jt} c_0} \left( b_{jt} - \frac{A_{jt-1}}{YP_{jt}} \right) \quad \text{where } c_0 = 5; c_1 = 0.02 \]

\[ r = \sqrt{1 - 2q + q^2 + c_1} \]

For the purpose of the trade-off, the marginal propensity to import \( m \) is approximated on the basis of assumptions about requirements for food and raw materials, energy, manufactures and services described in previous sections:

\[ m_{jt} = \left( c_0 PA_t (AD_{jt} - AH_{jt}) + c_1 PE_t (ED_{jt} - EH_{jt}) + MM_{jt} + MS_{jt} \right) / Y_{jt} \]

where \( c_0 = 0.05 \) and \( c_1 \) is the base year price of oil.

Finally, to complete the determination of domestic income, note that the level of domestic expenditure consistent with the capacity utilization target modelled in the Appendix is given by

\[ H_{jt}^* = YP_{jt} U_{jt}^* - \frac{1}{RX_{jt-1}} \frac{YP_{jt}}{YP_{jt-1}} \left( B_{jt-1} - m_{jt} \left( U_{jt}^* - U_{jt-1} \right) \right) \]

Of course, the above elaboration presupposes that there is no coordinated policy intervention other than that aimed at allowing governments some discretion in their responses to the domestic and external context. These contexts limit the extent to which individual governments are able to bring national economies near to desired targets for growth, stability and employment. In a different framework based on concerted policy
measures at the international level, many other possibilities can be imagined. For example, output potential could rise in response to official development assistance and foreign direct investment, population movements and improved resource management; or demand could shift to the benefit of poor, deficit countries by changes in the exchange rate, trade preferences, or subsidies and tariffs. In analytical terms, what has been modelled in terms of fixed functional forms and autocorrelated residuals may have to be re-specified in order to incorporate policy coordination. Extended models enhanced in this way could be used to gauge the potential for policy action beyond the constraints imposed by the modelling associated with the dominant free-market paradigm.

5.19 Households, firms and government

Given the policy context described in the previous section, domestic accumulation of financial assets and debt is determined by the behaviour of households and firms, with government finance as a residual element. This analysis is necessary in order to provide a framework for the management of public debt.

**BLOC VARIABLES**

- $PK_{jt}$: price of tangible assets (exogenous, index)
- $IH_{jt}$: household purchases of tangible assets (less sales & depreciation, PPP)
- $KH_{jt}$: tangible assets held by households (PPP)
- $DH_{jt}$: non-equity household domestic currency financial assets (net, PPP)
- $DC_{jt}$: non-equity domestic currency financial assets of firms (net, PPP)
- $DG_{jt}$: government’s domestic currency debt (net, PPP)

Household borrowing is linked to the real estate market while the net financial position of firms is usually nearly balanced.

\[
IH_{jt} = c_0 \left( KH_{j,t-1} - KH_{j,t-2} \right) + Y_{jt} \left( b_j + u_{jt} \right) \quad \text{where } c_0 = 0.2
\]

\[
KH_{jt} = \frac{PK_{jt}}{PK_{j,t-1}} KH_{j,t-1} + IH_{jt}
\]

\[
DH_{jt} = Y_{jt} \left( b_j + u_{jt} \right) - c_0 KH_{jt} \quad \text{where } c_0 = 0.5
\]

\[
DC_{jt} = Y_{jt} \left( b_j + u_{jt} \right)
\]

Fiscal policy must be implemented in a manner that is consistent with private saving and borrowing. In particular, government debt provides the ultimate source for domestic bonds and deposits:

\[
DG_{jt} = \frac{DF_{jt}}{RX_{jt}} - DH_{jt} - DC_{jt}
\]
APPENDIX
DERIVATION OF THE MONETARY POLICY TRADE-OFF

NOTATION

\[ Y \] expected or actual level of income
\[ YP \] productive potential or full employment level of income
\[ U \] expected or actual capacity utilization
\[ B \] expected or actual current account
\[ H \] expected domestic expenditure on goods and services
\[ R \] improvement in external asset position (over the minimum requirement)

Given a target for capacity utilization \( U \), outcomes may be forecast as follows:

\[
Y = U \cdot YP \\
B / YP = B_{-1} / YP_{-1} - m(U - U_{-1}) \\
H = Y - B
\]

where

\[ m \] is the marginal propensity to import.

The minimum objective for the balance of payments current account in the current year will be

\[
B_{\text{min}} = \left( b - A_{-1} / YP \right) / n
\]

where

\[ b \] is a long-run lower limit on net external assets relative to productive potential and
\[ n \] is the horizon over which the limit must be respected.

Assume that management of reserves gets priority when the expected balance of payments current account approaches the minimum level and demand management gets priority otherwise. This may be represented by an objective function

\[
V = p \ln(B / YP - B_{\text{min}}) - (1 - U)^2
\]

where \( p \) determines the relative importance given to each objective.
The first-order condition for an efficient trade-off is
\[
dV/dU = -p/(q-U) + 2(1-U) = 0
\]
where \( q = U_{-1} + B_{-1} / m \cdot YP_{-1} - (b - A_{-1} / YP)/(m \cdot n) \)

Rewriting this condition as a quadratic
\[
U^2 - (q + 1)U + (q - p/2) = 0
\]
we find that the maximum value of \( V \) is achieved when
\[
U = (1 + q - r)/2
\]
where \( r = \sqrt{1 - 2q + q^2 + 2p} \)
REFERENCES


Polak, J. 1957. ‘Monetary Analysis of income formation and payment problems’. IMF Staff Papers 6(1).


NOTES

1. UN-DESAP has facilitated access to data compiled by the U.N. Statistical Office. This work relies on intellectual contributions from many generations of colleagues at the University of Cambridge and elsewhere and, in particular, Wynne Godley’s work on the relation between aggregate demand and financial balances.

2. Dissenting voices, like Godley (1999), Baker, Epstein and Pollin (1998), Shiller (2000), Martin (2000) and others were ignored by default. Even A. Greenspan from his position as chairman of the US Federal Reserve had only managed to characterize the situation as ‘irrational exuberance’ but could not draw policy conclusions from there.

3. Few studies managed to underscore the fact that the success of many countries in such regions was in great part the result of a proactive role of the government in building the infrastructure and educational fabric as much as intervening directly in the most strategic sectors (Jomo, 1998; Wade, 1990).

4. See, for example, Papadimitriou et al, 2003, 2004. As noted in these studies, a revival of the fiscal solution to an extent not seen before would under other circumstances have faced strong opposition from free-marketers.

5. Although the length of a ‘period’ is formally undefined, the analysis and equations set out in this paper are intended for use in a modelling framework that relies on annual time series. The adjustment functions generally have one-period lag terms.


7. At this level of generality, the rate $\alpha$ encompasses changes in asset prices, exchange rates, etc. that affect the end-period value of the portfolio. If the accounting is performed in ‘real’ terms, holding gains will be measured relative to the rate of inflation.

8. The main source of holding gains on tangible assets is real estate.

9. For practical modeling purposes, countries may be aggregated, the world as a whole being represented by anywhere between 10 and 80 distinct countries or country groups.

10. Readers may be more familiar with an expression of the form $H_t = I_t + (1 - s)Y^*_t$. This formulation does not draw attention to the relationship between savings and growth of income implied by a wealth target. The adjustment process and interdependence of income, expenditure and accumulation of financial assets were set out in Godley and Cripps (1983), chapter 3. For a country as a whole, expression (8) carries the implication that growth of expenditure relative to income is influenced by the desired net acquisition of financial assets (i.e., the financial component of the balance of payments). The influence of holding gains on the adjustment process is an open question that will be addressed empirically.

11. The interpretation of such a relationship is subject to different views. Polak (1957, 1995), in what was seen as the ‘official’ view of Breton Woods institutions, argues that the causality runs from foreign inflows to the money supply and thence inflation. Price stability would result from fiscal discipline. Authors such as Hausmann et al. (1996) see price volatility resulting from a mix of domestic factors (demand, policy regimes, etc) and external factors (trade shocks, international liquidity, etc). Accordingly, even if there is no single prescription, it is understood that countries could insulate themselves by greater openness and financial integration. Calvo and Reinhart (2000) argue that the stability of prices, interest rates and exchange rates is not only related with international reserves but also with credibility and investors’ perception of the ‘country’s fundamentals’. Even if the authors did not provide recommendations, they acknowledged that the existence of such uncertainties might justify a degree of policy intervention. For authors in the U.N. Economic Commissions for Latin America and Africa (ECLAC, ECA), the emphasis was on external factors and thus policy would need to act counter-cyclically to avoid lasting impact on employment and income.

12. The paradigmatic example is the Euro Area under the Growth and Stability Pact. See, for example, Bibow and Terzi (2007, forthcoming) and Irvin (2006). An analysis of macroeconomic policy and the ‘Golden Rule’ in the United Kingdom can be found in Godley and Izuirieta (2003).

13. It is generally expected that the interest rate instrument alone would suffice to trigger the necessary shift of preferences of financial investors. Yet, interest rates are not the only factor driving portfolio decisions. Furthermore, there is a limit to fine tuning of interest rates because they cannot be negative (Kaldor, 1939).

14. Research work as early as that of Mundell (1960, 1961) and McKinnon (1963) concluded that the potential advantages of exchange rate flexibility are not universal, citing issues such as size, openness and labour mobility. During recent decades, the question was revisited theoretically and empirically. Except during the 1980s and early 1990s, when Breton Woods institutions enforced a policy-shift towards external liberalization, most studies agreed that full external liberalization for low-income countries is problematic at best (Krugman and Taylor, 1978; Vos, 1995; Ocampo and Taylor, 1998; Gauzu et al. 2000; Taylor, 2001)

15. For an analysis of the structure of aggregate demand, lending and sustainability of the U.S. macro-financial imbalances, see, for example, Godley (1999), Godley and Izuirieta (2001) and Cripps et al. (2005).

16. For example, table B.100 of the FED’s Flow of Fund Accounts of the U.S. (Federal Reserve 2006) shows that since the early 1990s the net worth of the personal sector has shifted from a historic average of four and half times disposable income to nearly six times. As a result, estimates of holding gains during recent years were around one trillion dollars (about 15 per cent of disposable income of the personal sector), as recorded in table R.100.

17. See, for example, Izuirieta (2005a) for the analysis of holding gains in the U.S. and the impact on private sector spending and aggregate demand, and Muellerbauer (2003) for a similar analysis for Europe, in which emphasis is given to the differentiated impact on aggregate demand depending on lending institutions.
18. See Bernanke (2005): “Some observers have expressed concern about rising levels of household debt, and we at the Federal Reserve follow these developments closely. However, concerns about debt growth should be allayed by the fact that household assets (particularly housing wealth) have risen even more quickly than household liabilities.” Similar remarks can be found in Greenspan (2005).

19. Recent evidence from the US seems to suggest a direct impact of stock market appreciation on consumption in the short run (Dynan and Maki, 2001; Case et al. 2005) but the ‘identification problem’ noted in the text remains an issue. Finally, it is unclear whether the effects of changes of investment position due to revaluation of assets and liabilities are symmetric.

20. Estimates of the international investment position of the US at year end 2005 show that the change in net position due to holding gains alone was a positive $ 700 billion (about 6 per cent of U.S. GDP or two per cent of global GDP) while the current account was a negative $ 800 billion. The net gain comprised a $ 1,080 billion gain on U.S. resident assets in stock markets abroad and a $ 380 billion loss due to dollar appreciation. During the years 2002-2004, positive holding gains on both counts (appreciation of external stock market prices and dollar depreciation) almost fully compensated for the current account deficits (Oxford Analytica, 2006). Thus, the net liability position of the U.S. has remained broadly unaltered in the last five years (about 20 per cent of GDP) despite the fact that annual net increments have averaged 4.5 per cent of GDP (the counterpart of the current account deficit). The net liability position would have reached more than 40 per cent of GDP by end 2005 in the absence of such favourable asset and liability revaluations.


22. In 1857 Ernst Engel indicated that as household income increases, the proportion spent on food decreases. Such a relation has since been corroborated. On the occasion of the centennial of Engel’s finding, Houthakker (1957) found that the correlation persists among a wide range of countries, domestic regions and epochs. As shown by many authors, Engel’s law is at the core of what is known as the Prebisch-Singer hypothesis (Prebisch 1950; Singer, 1950), namely, that the deterioration over time of the relative price of primary commodity exports vis-à-vis manufactured exports. Such a relation is clearly observable in historic series of food and raw materials.

23. The imputed unit elasticity might run counter to the assumption of flexible technological adaptation that is attributed to the development process. As noted by Wuyts (1998, pp. 30), “in per capita terms CO2 emissions (and energy consumption) remain much higher in the developed world than in developing countries”. Cross-section plots of per capita energy use and income by country show an elasticity around unity.

24. If there is a sizeable group of international speculators, the exchange rate will settle at the value that the speculators believe to be appropriate. Anything else causes them to move in or out of the given currency. The monetary authority can at best influence judgments made by speculators (e.g., “talking down the dollar”).

25. This deflator should be denominated in domestic currency terms to measure erosion of the value of domestic currency assets and liabilities due to inflation.

26. The equivalent expression using rates of inflation is

\[ GA = \left( -\frac{\hat{m}}{1 + \hat{m}} \right) \frac{FF_{-1}}{1 + \hat{p}} - \frac{\hat{x} - \hat{p}}{1 + \hat{p}} FH_{-1} + \frac{\hat{q} - \hat{m}}{1 + \hat{m}} QF_{-1} - \frac{\hat{d} - \hat{m}}{1 + \hat{m}} QH_{-1} \]

where

\[ \hat{p} = PH / PH_{-1} - 1; \quad \hat{m} = PXM / PXM_{-1} - 1; \quad \hat{d} = PD / PD_{-1} - 1; \quad \hat{q} = PQ / PQ_{-1} - 1; \quad \hat{x} = RX / RX_{-1} - 1 \]